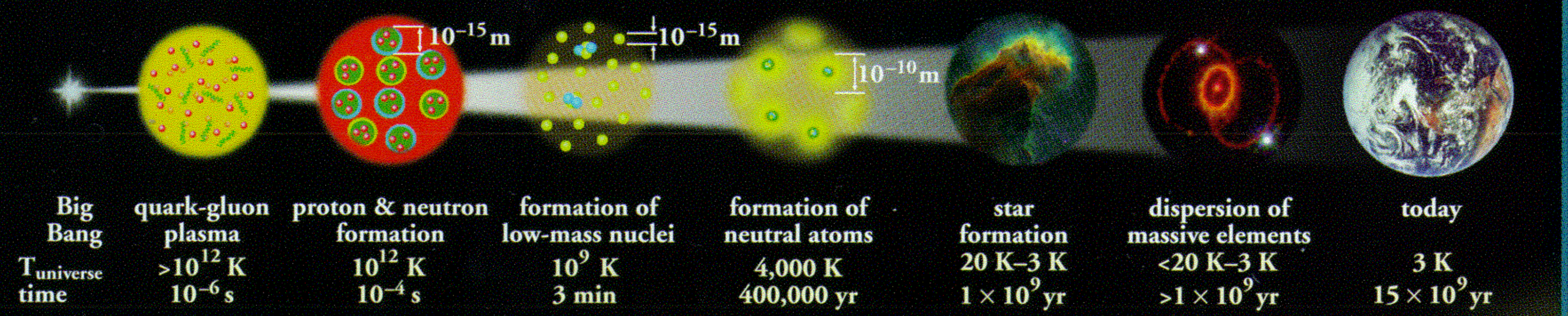
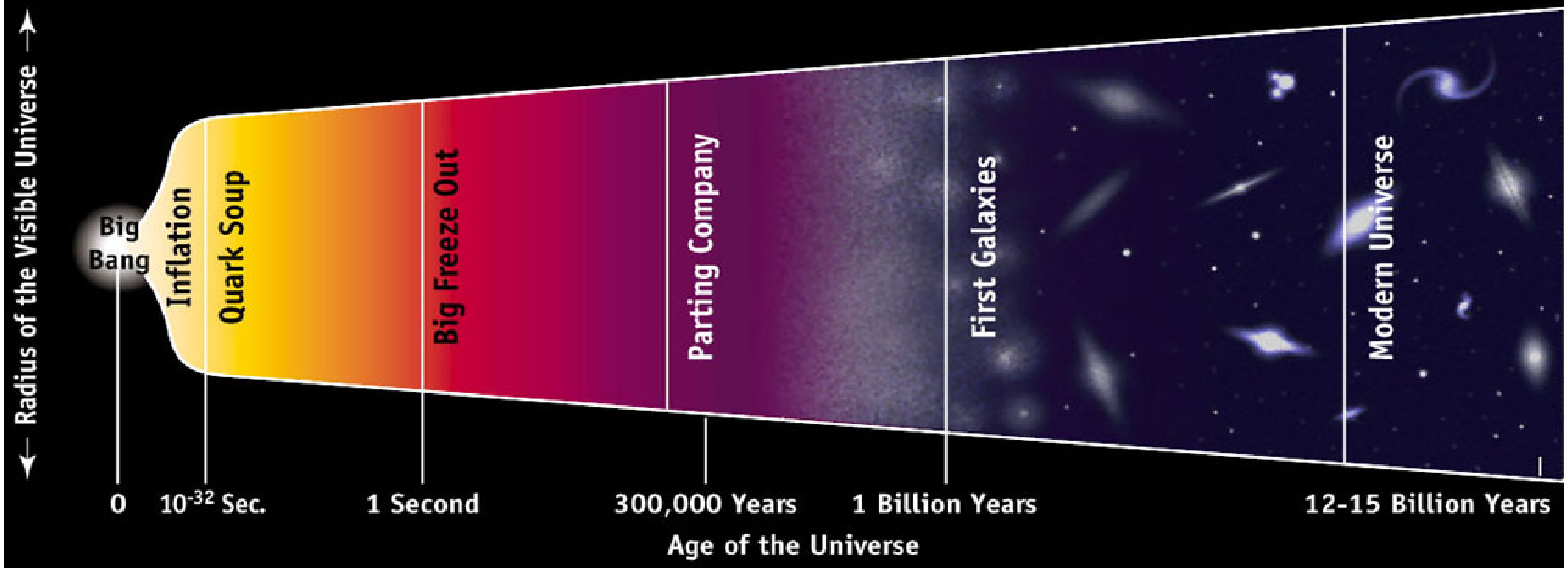
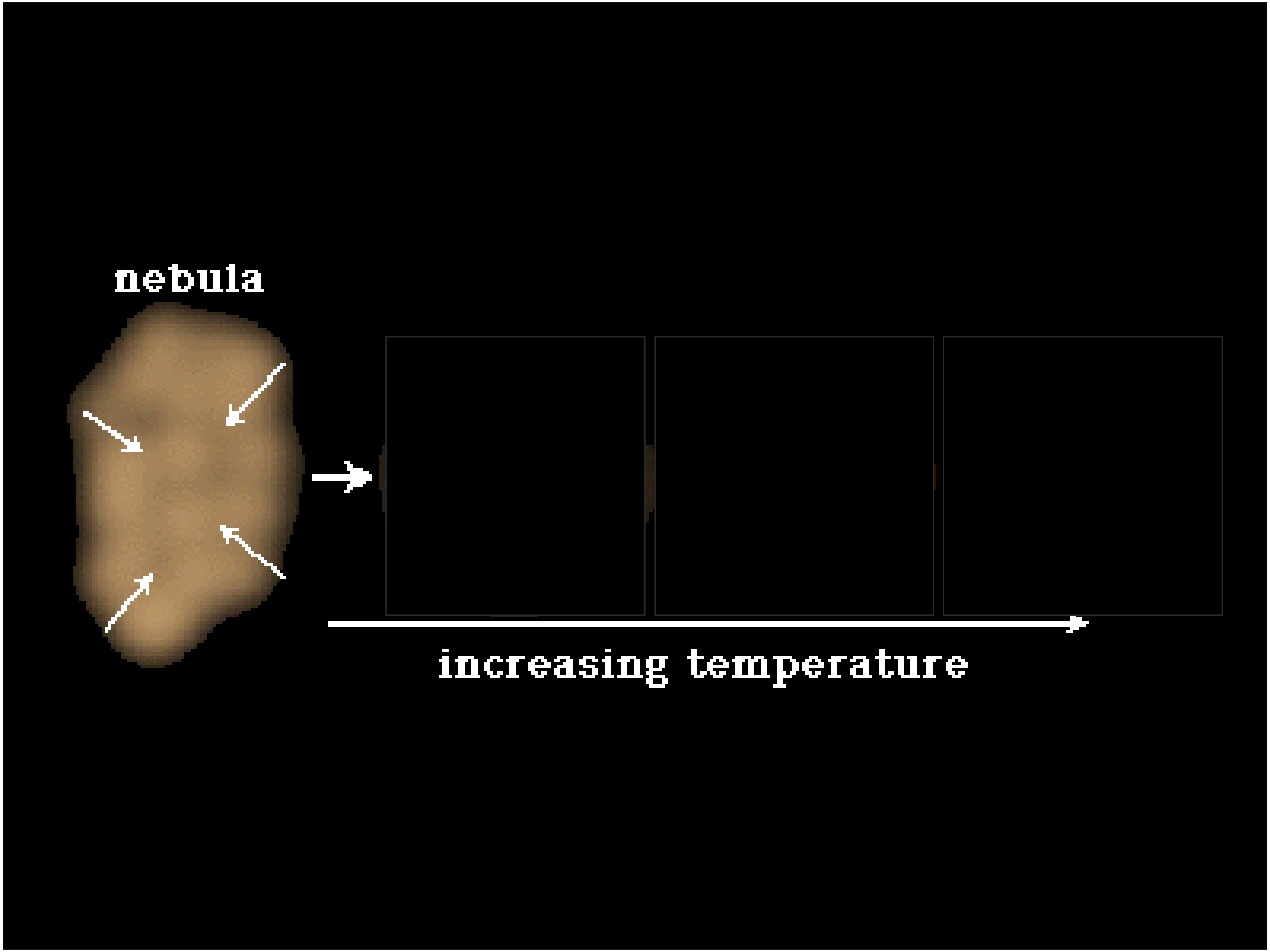
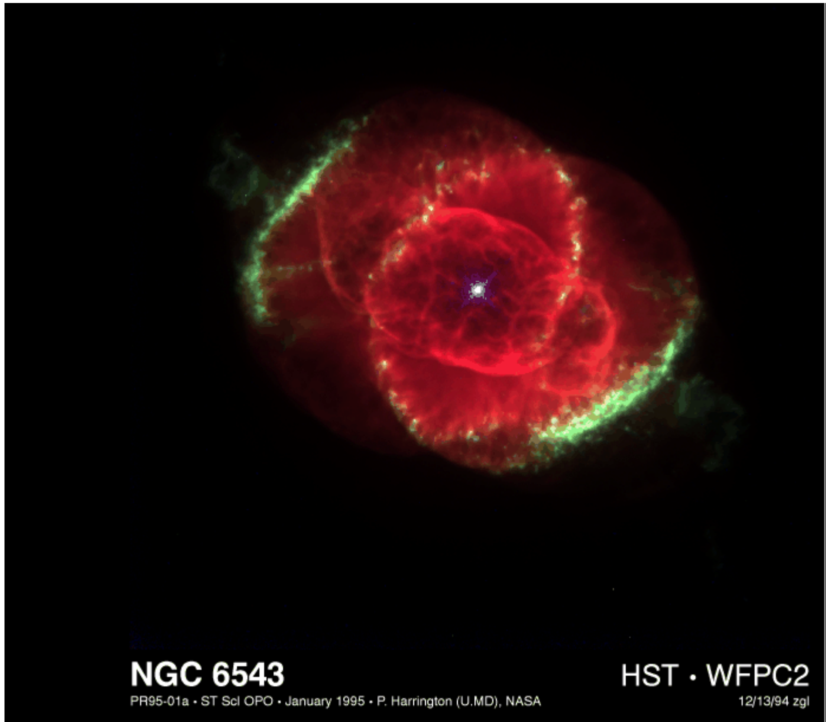
Expansion of the Universe

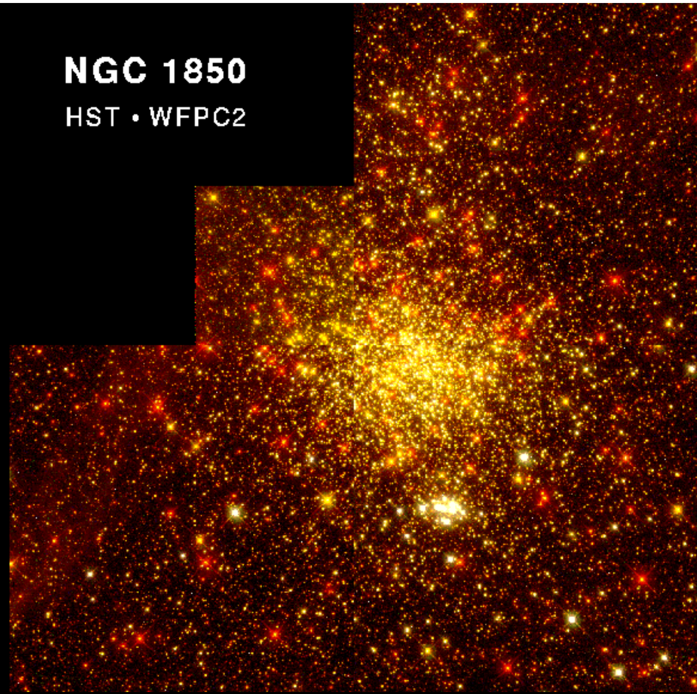
After the Big Bang, the universe expanded and cooled. At about 10^{-6} second, the universe consisted of a soup of quarks, gluons, electrons, and neutrinos. When the temperature of the Universe, $T_{universe}$, cooled to about 10^{12} K, this soup coalesced into protons, neutrons, and electrons. As time progressed, some of the protons and neutrons formed deuterium, helium, and lithium nuclei. Still later, electrons combined with protons and these low-mass nuclei to form neutral atoms. Due to gravity, clouds of atoms contracted into stars, where hydrogen and helium fused into more massive chemical elements. Exploding stars (supernovae) form the most massive elements and disperse them into space. Our earth was formed from supernova debris.

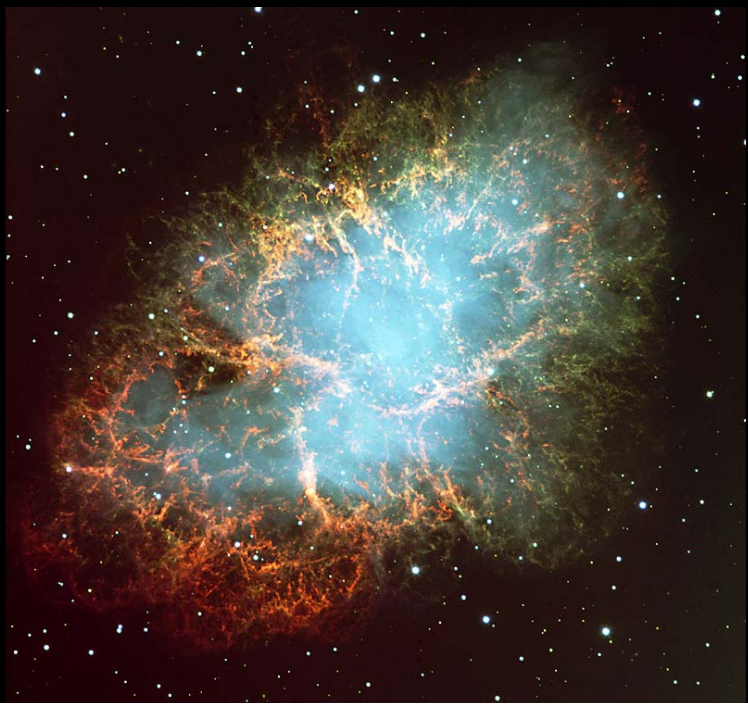





nebula
increasing temperature







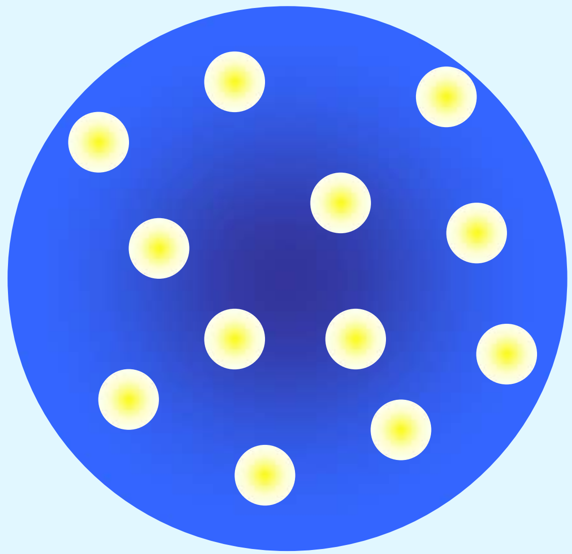
Supernova 1987A Rings

Hubble Space Telescope

NUCLEO DE THOMSON

ANIMAÇÃO

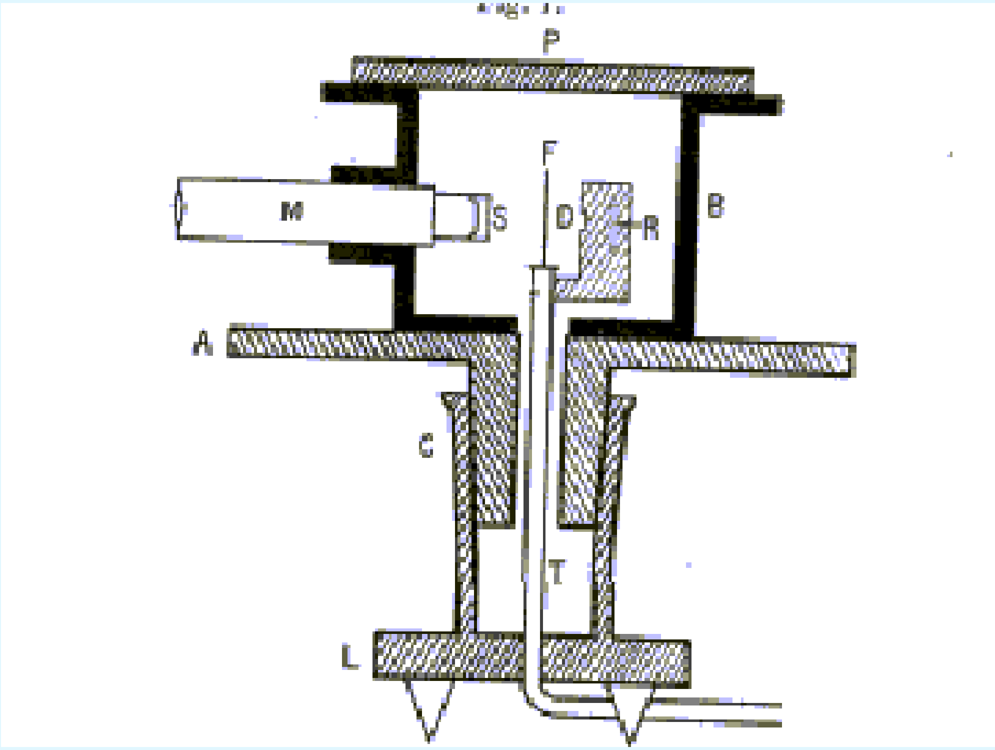
http://galileoandeinstein.physics.virginia.edu/more_stuff/Applets/rutherford/rutherford2.html



^{24}Mg

NUCLEO DE RUTHERFORD

ANIMAÇÃO



http://galileoandeinstein.physics.virginia.edu/more_stuff/Applets/rutherford/rutherford.html

http://www.nat.vu.nl/~pwgroen/projects/sdm/applets.htm

http://www.nat.vu.nl/~pwgroen/sdm/hyper/anim/anim_DI.html

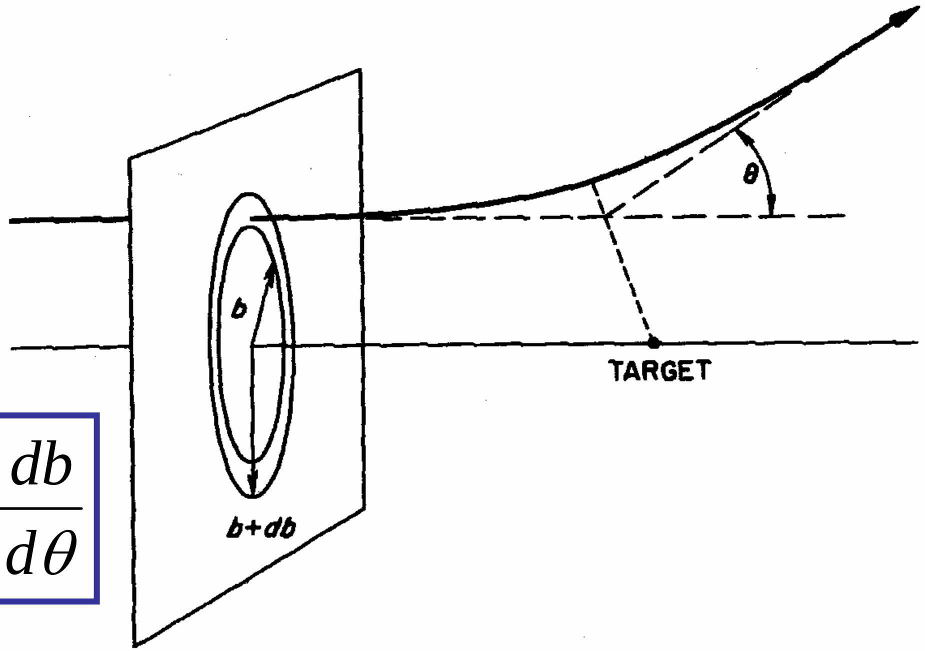
http://physics.uwstout.edu/physapplets/

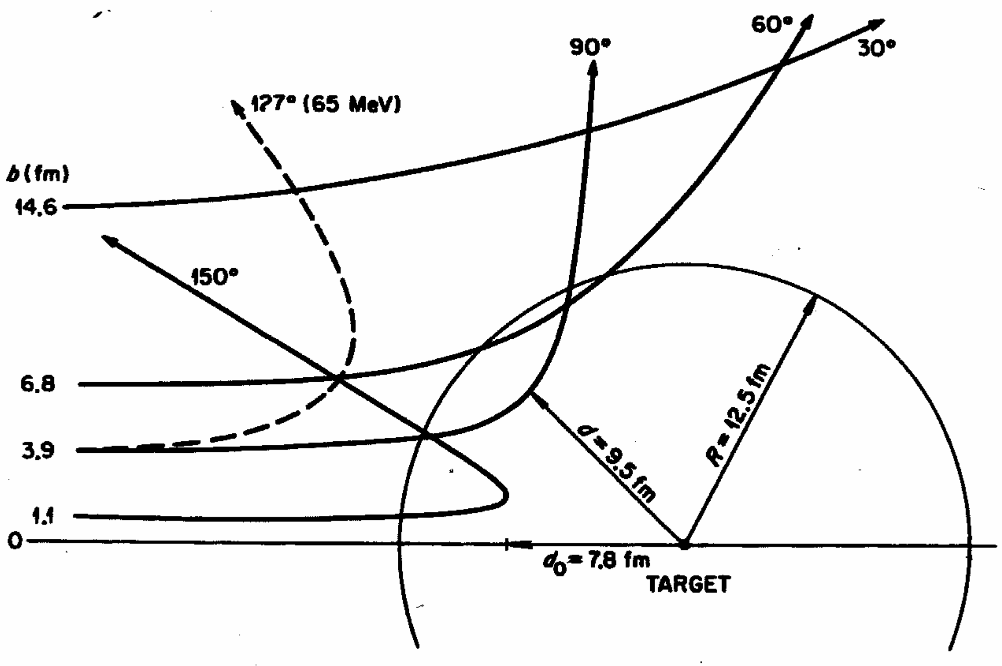
http://micro.magnet.fsu.edu/electromag/java/rutherford/

$$\frac{d\sigma}{d\theta} = \frac{d\left(\pi b^2\right)}{d\theta} = 2\pi b \frac{db}{d\theta}$$

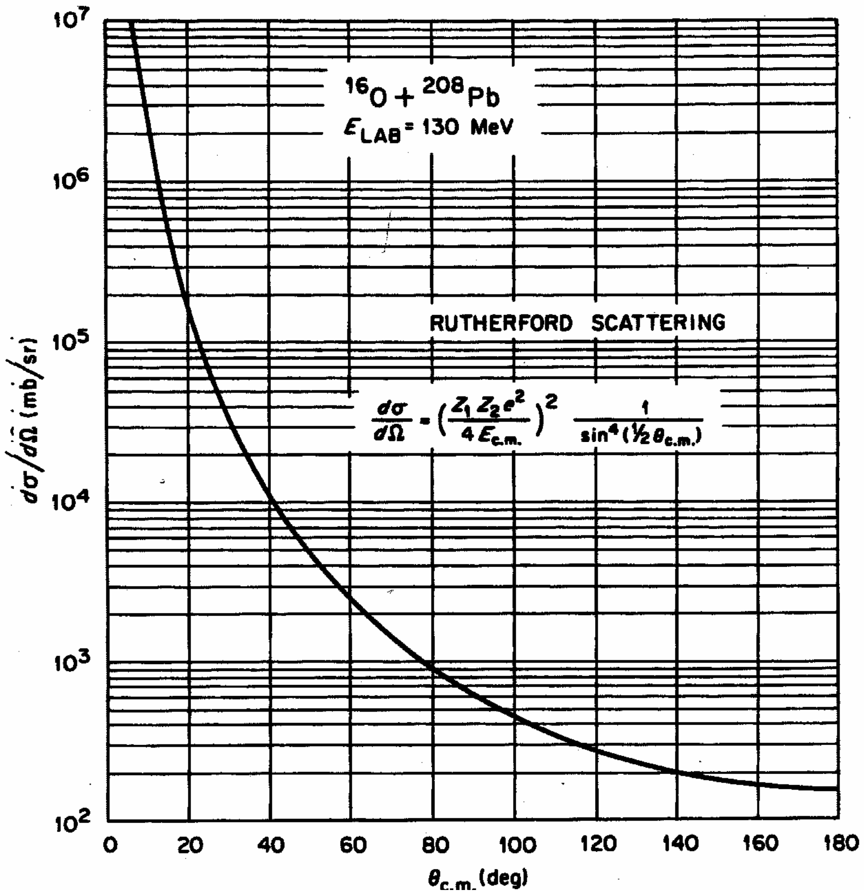
$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\theta}\left(\frac{d\theta}{d\Omega}\right)$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{2\pi\,\mathrm{sen}\,\theta}\right)\left(2\pi b \frac{db}{d\theta}\right) = \frac{b}{\mathrm{sen}\,\theta}\frac{db}{d\theta}$$





$$\frac{d\sigma}{d\Omega}=\left[\frac{Z_A Z_a e^2}{2E}\right]^2\frac{1}{\mathrm{sen}^4\frac{\theta}{2}}$$



espalhamento:

^{208}Pb(^{16}O,^{16}O)^{208}Pb

na energia $\mathbf{E}_{\mathbf{^{16}O}}$= 130MeV

a curva pontilhada corresponde a $E_{(16O)}$ = 65MeV

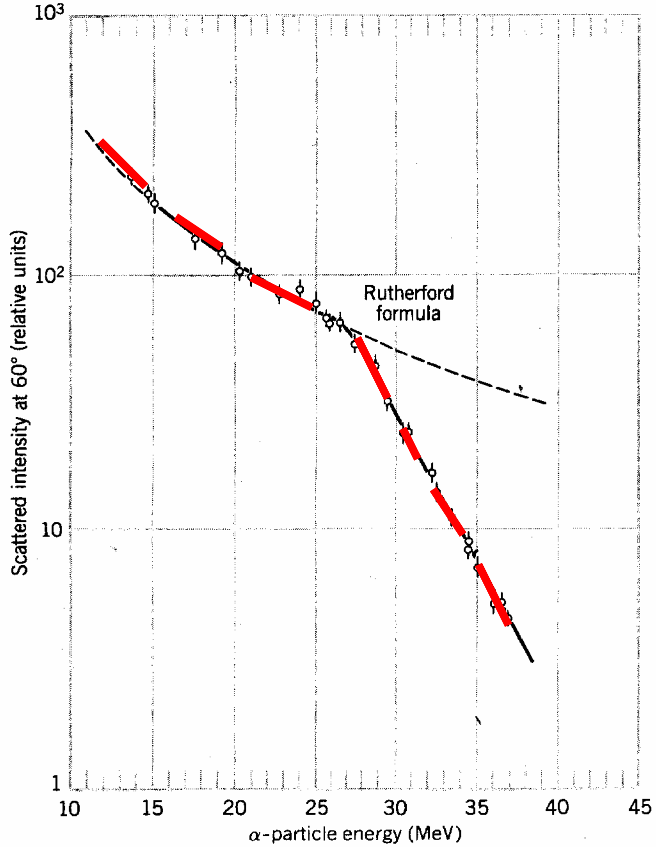


Figure 3.11 The breakdown of the Rutherford scattering formula. When the incident α particle gets close enough to the target Pb nucleus so that they can interact through the nuclear force (in addition to the Coulomb force that acts when they are far apart) the Rutherford formula no longer holds. The point at which this breakdown occurs gives a measure of the size of the nucleus. Adapted from a review of α particle scattering by R. M. Eisberg and C. E. Porter, *Rev. Mod. Phys.* **33**, 190 (1961).

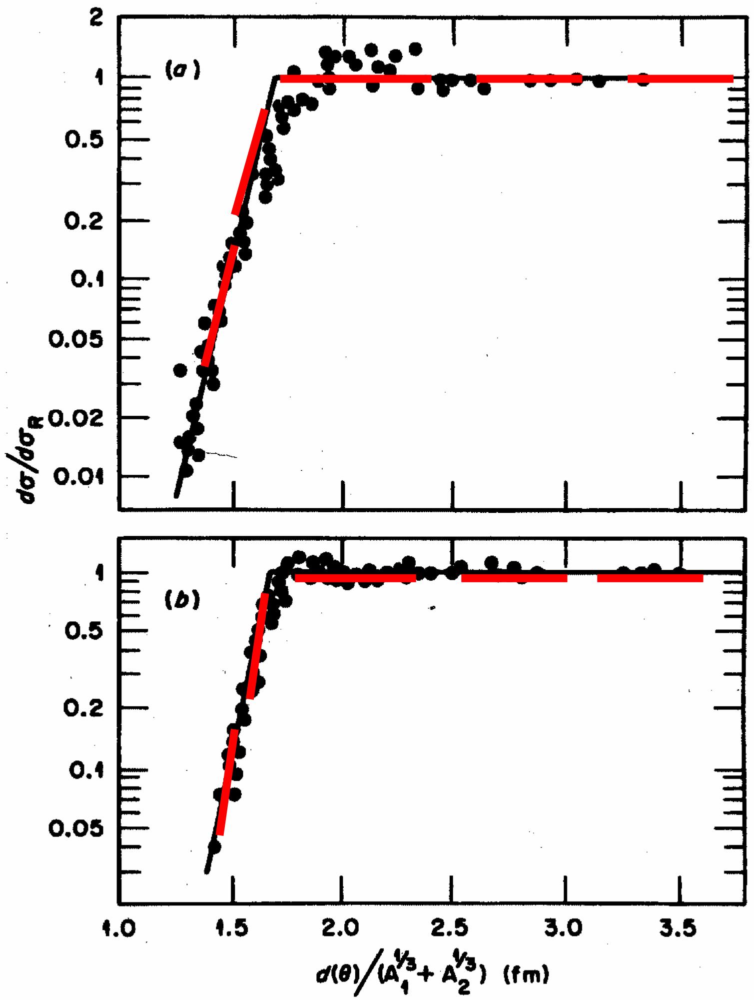
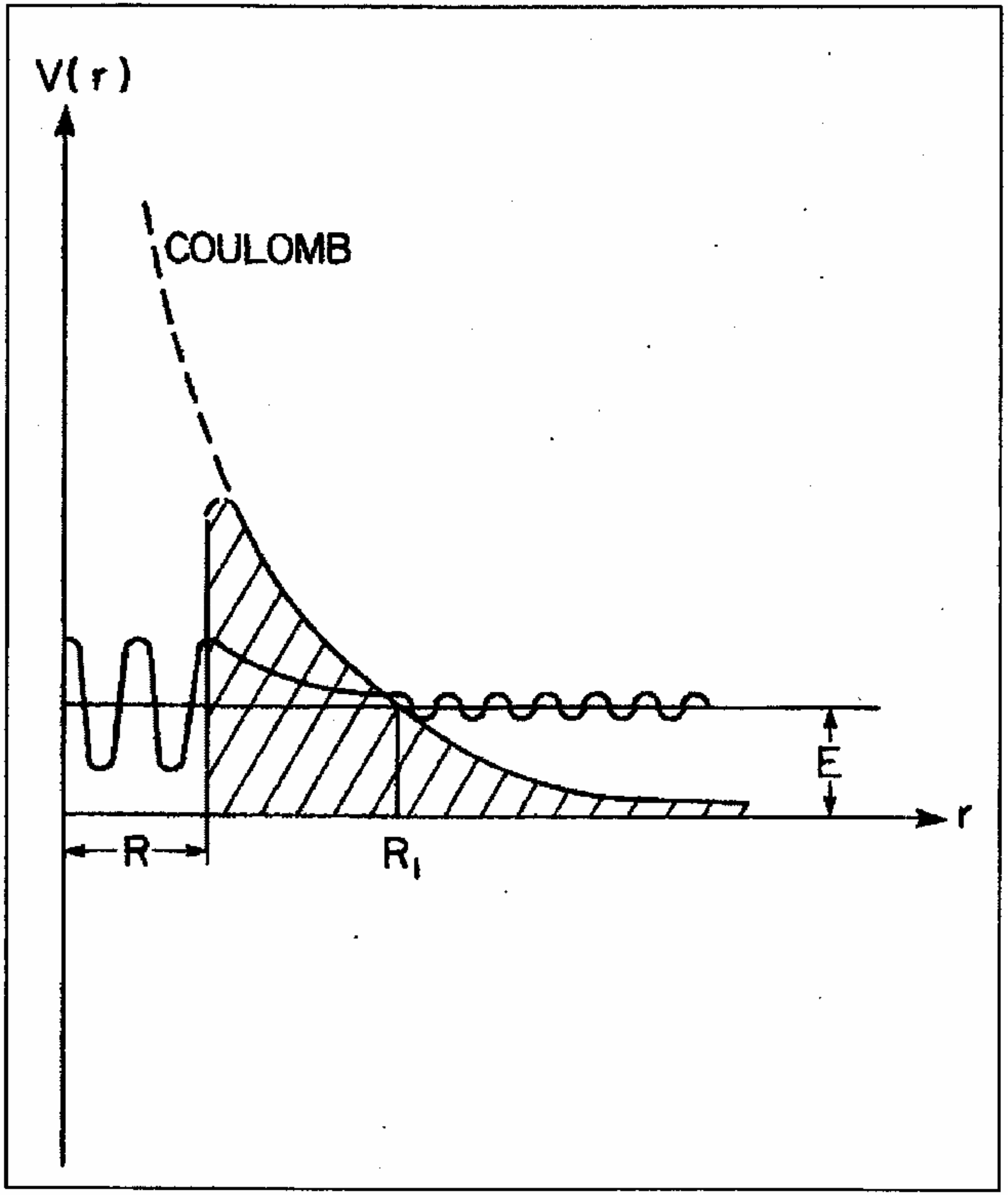
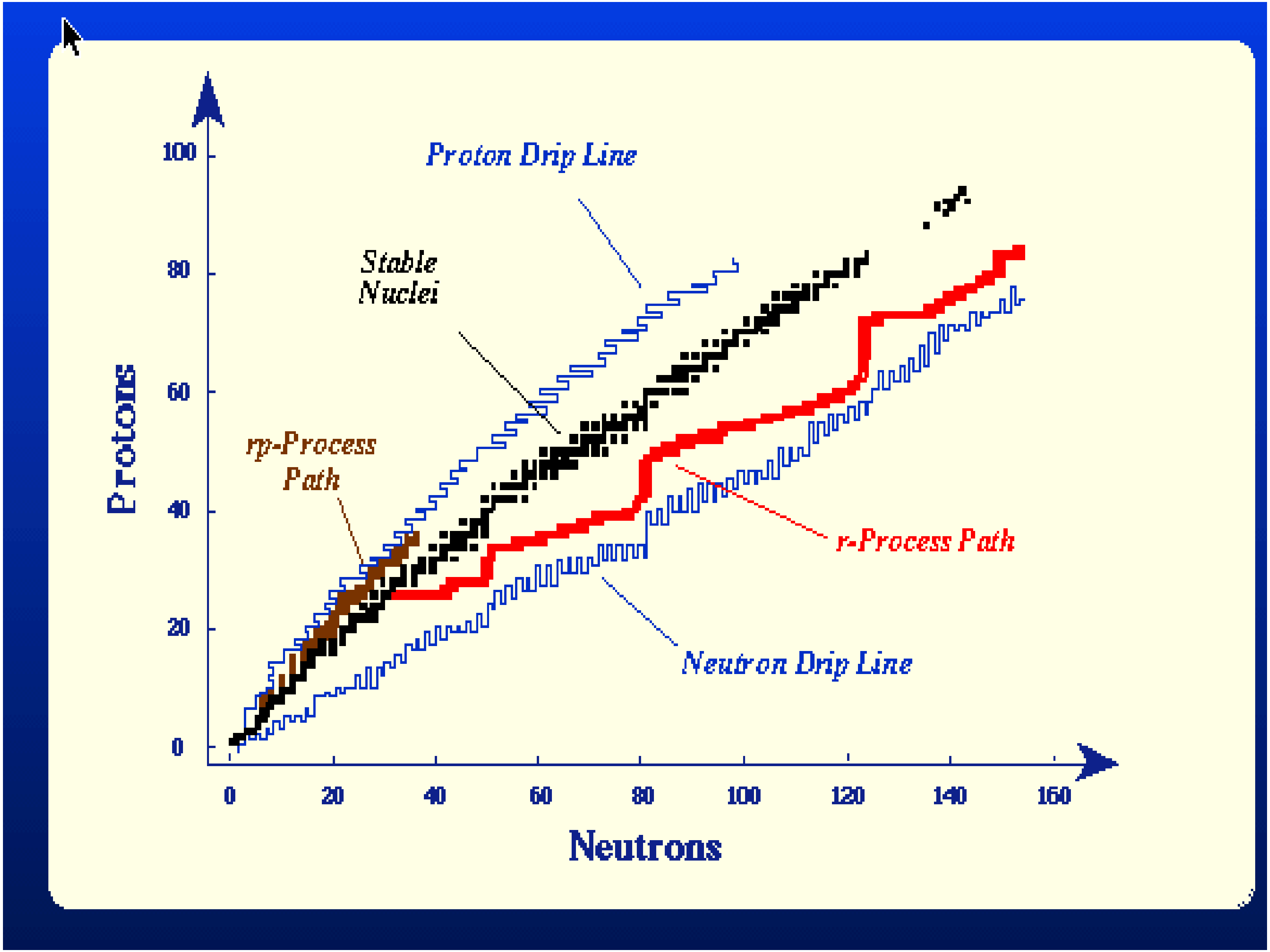
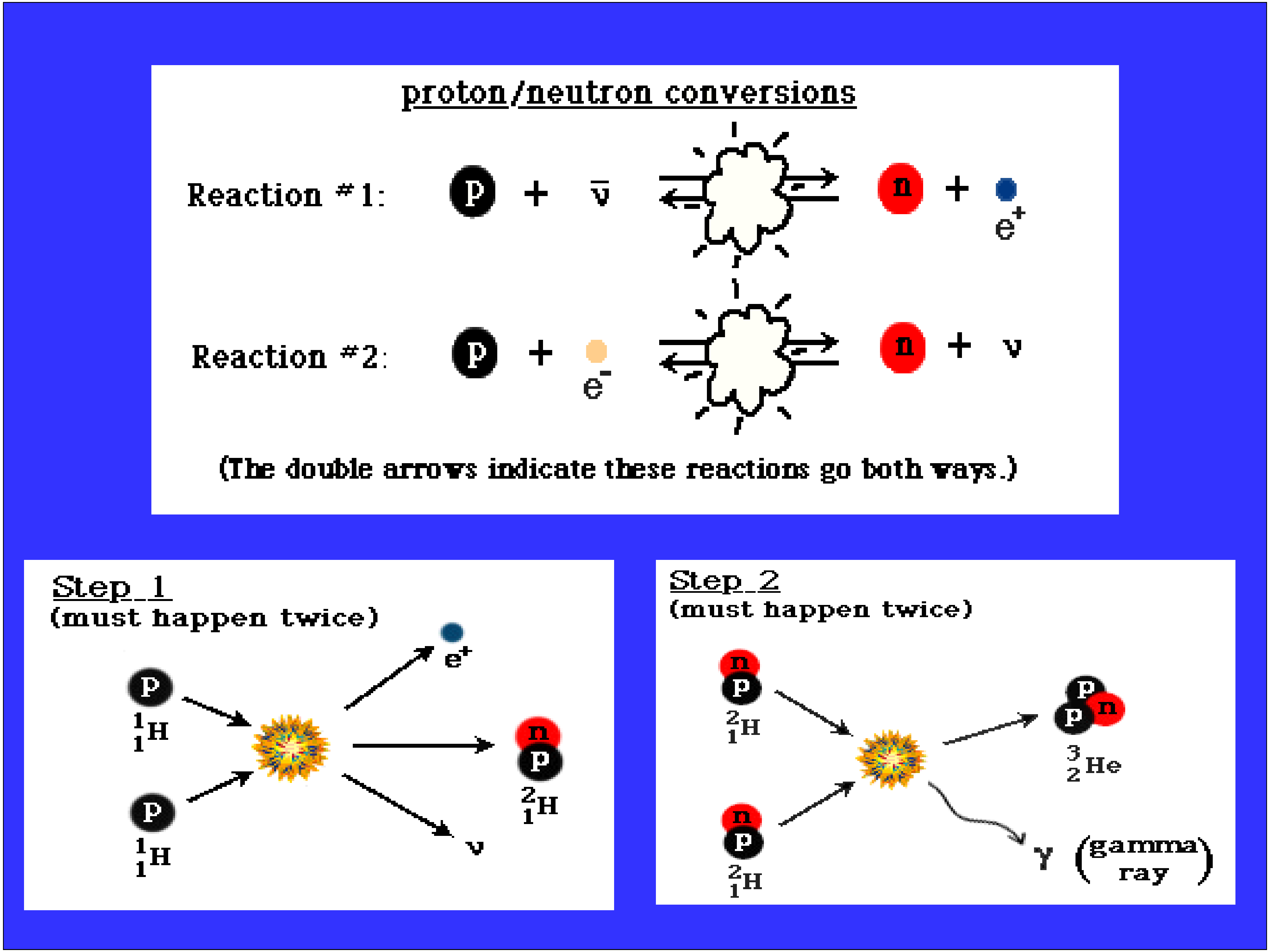


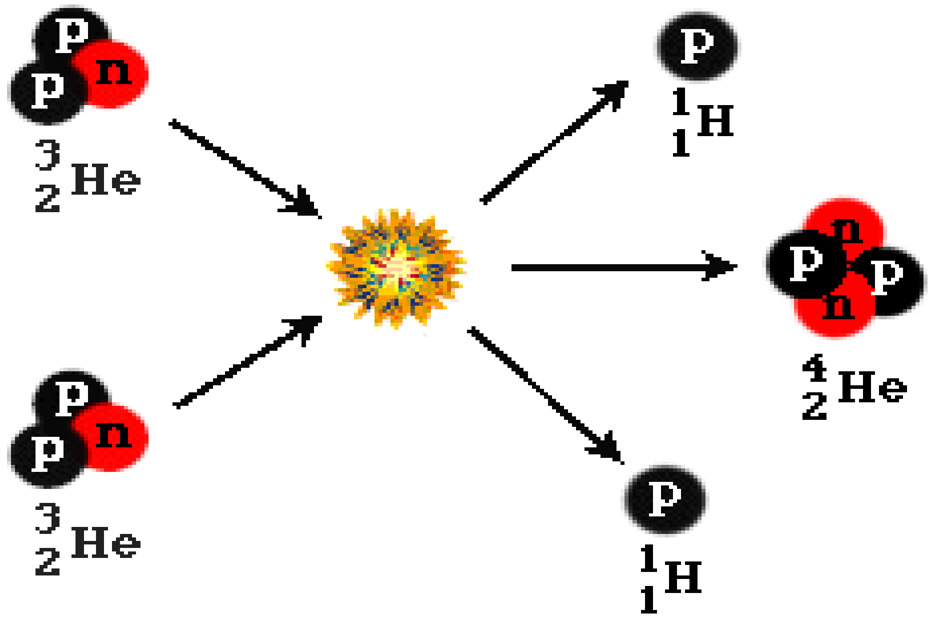
Figure 2.12 The differential cross-sections for O and C ions, in ratio to the Rutherford ones, scattering from various targets plotted against the distance of closest approach d, instead of the scattering angle θ, by using equation 2.20. The distance d has been divided by $(A_1^{1/3}+A_2^{1/3})$, where A_i is the mass number of nucleus i. The measured cross-sections then fall on a universal curve, showing that nuclear radii are approximately proportional to $A^{1/3}$. (a) ^{16}O + ^{40,48}Ca at 49 MeV, ^{16}O + ^{40,48}Ca, ^{50}Ti, ^{52}Cr, ^{54}Fe, ^{62}Ni at 60 MeV and ^{18}O + ^{60}Ni at 60 MeV; (b) ^{12}C + ^{96}Zr at 38 MeV, ^{16}O + ^{96}Zr at 47, 49 MeV, ^{16}O + ^{88}Sr, ^{93}Zr at 60 MeV and ^{18}O + ^{90}Zr at 60, 66 MeV. (After Christensen *et al.*, 1973)


V(r)
COULOMB
E
r
R
R1

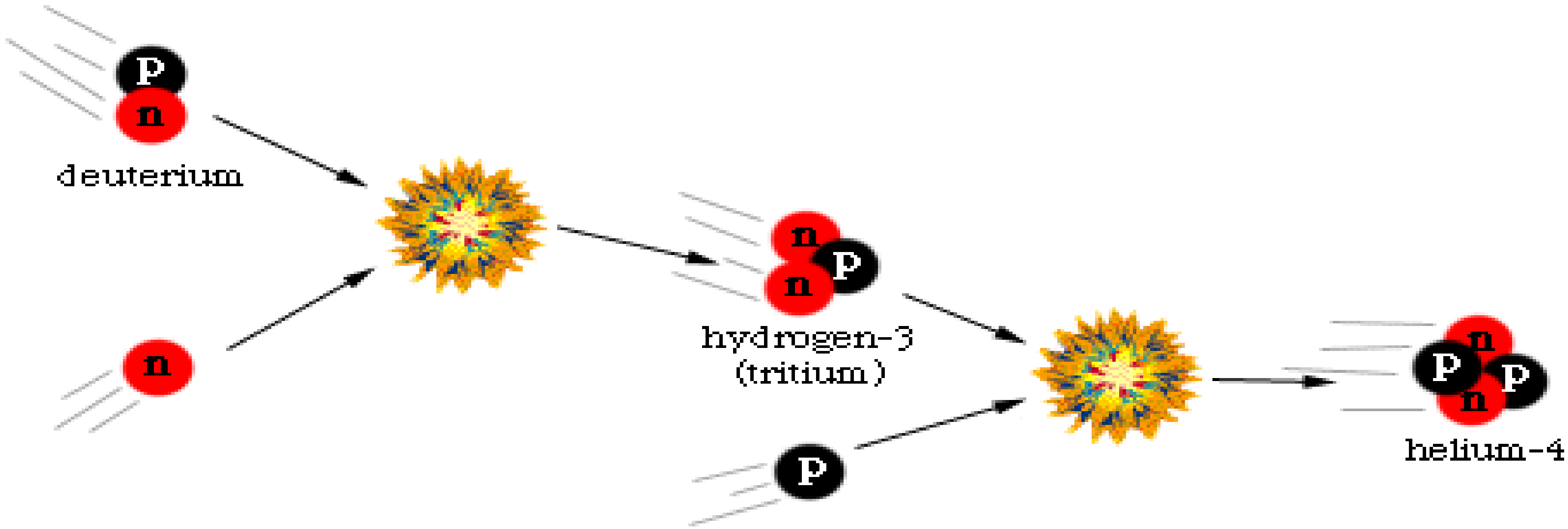

Proton Drip Line
Stable Nuclei
rp-Process Path
r-Process Path
Neutron Drip Line
Protons
Neutrons
0
20
40
60
80
100
0
20
40
60
80
100
120
140
160


proton/neutron conversions
Reaction #1:
p + ν̄
n + e+
Reaction #2:
p + e-
n + ν
(The double arrows indicate these reactions go both ways.)
Step 1
(must happen twice)
p
1 1 H
p
1 1 H
e+
n p
2 1 H
ν
Step 2
(must happen twice)
n p
2 1 H
n p
2 1 H
p p n
3 2 He
γ (gamma ray)

Step 3:



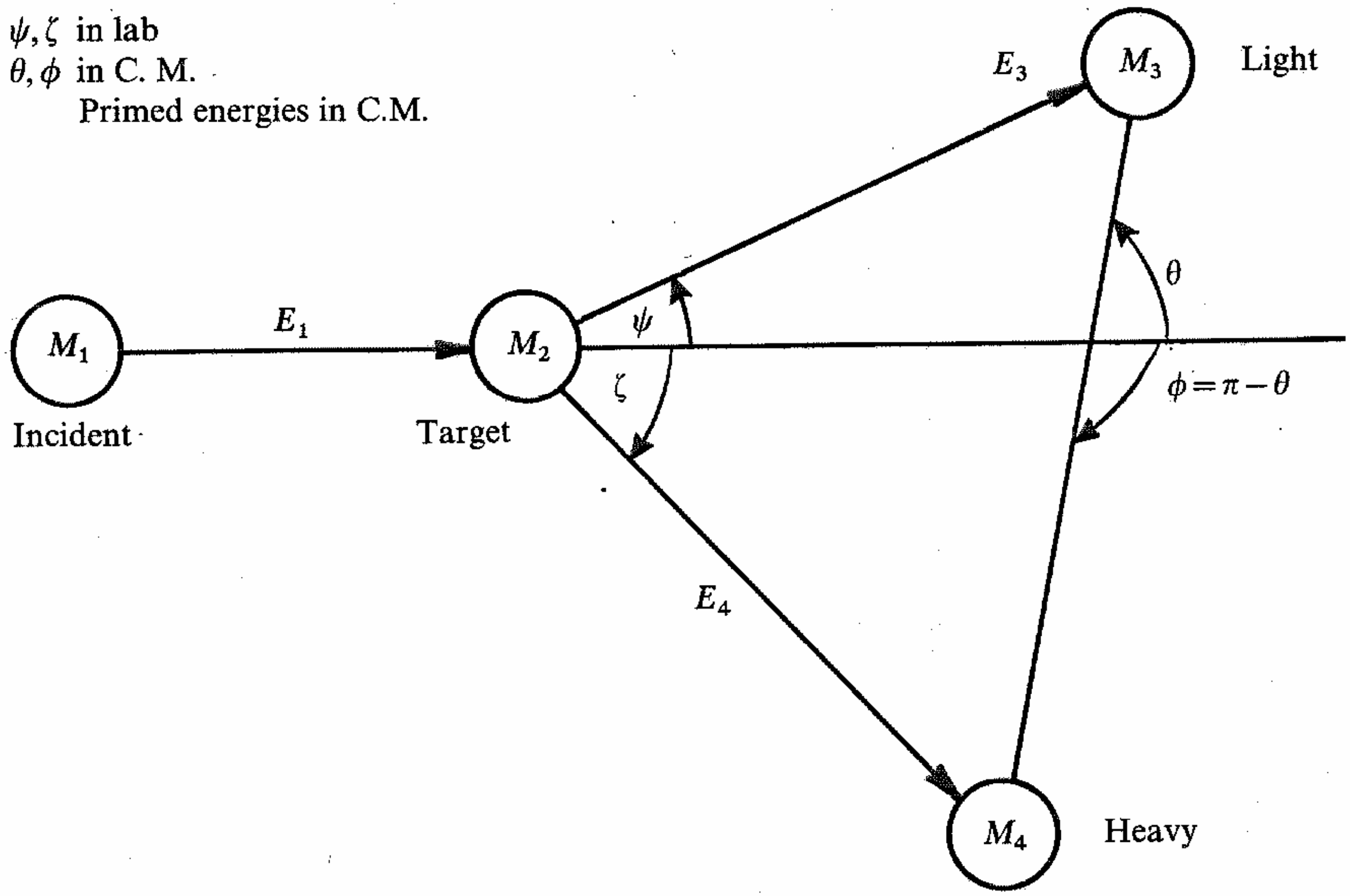
p
n
p
$^{3}_{2}He$
p
n
p
$^{3}_{2}He$
p
$^{1}_{1}H$
n
p
p
n
$^{4}_{2}He$
p
$^{1}_{1}H$



p
n
deuterium
n
n
p
n
hydrogen-3
(tritium)
p
n
p
p
n
helium-4

Kinematics of nuclear reactions and scattering (continued)

ψ, ζ in lab
θ, ϕ in C. M.
Primed energies in C.M.



$$Q=(M_1+M_2-M_3-M_4)c^2$$
$$E_T=E_1+Q=E_3+E_4$$

Define:

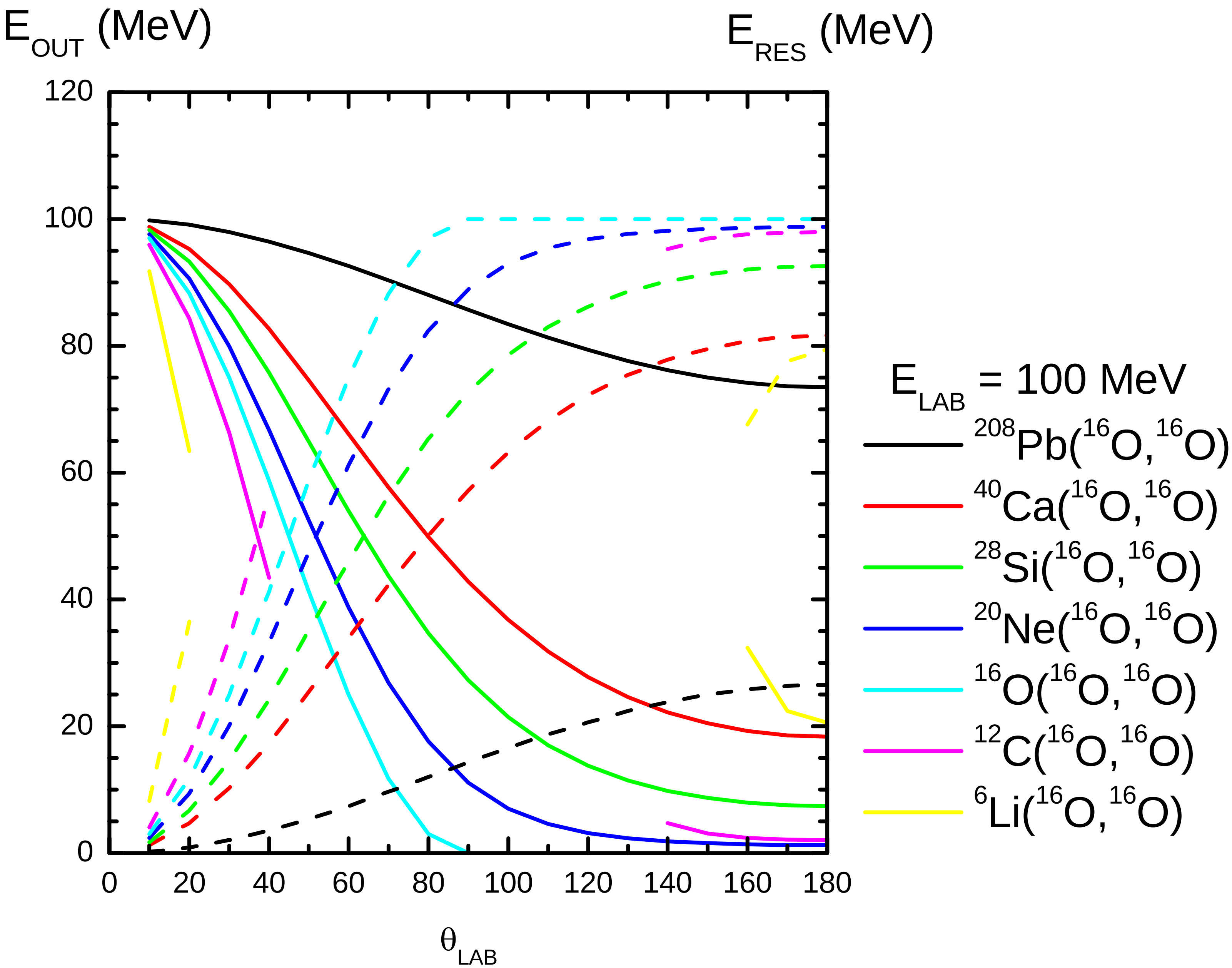
$$A=\frac{M_1M_4(E_1/E_T)}{(M_1+M_2)(M_3+M_4)}, \qquad C=\frac{M_2M_3}{(M_1+M_2)(M_3+M_4)}\left(1+\frac{M_1Q}{M_2E_T}\right)=\frac{E'_4}{E_T}$$

$$B=\frac{M_1M_3(E_1/E_T)}{(M_1+M_2)(M_3+M_4)}, \qquad D=\frac{M_2M_4}{(M_1+M_2)(M_3+M_4)}\left(1+\frac{M_1Q}{M_2E_T}\right)=\frac{E'_3}{E_T}$$

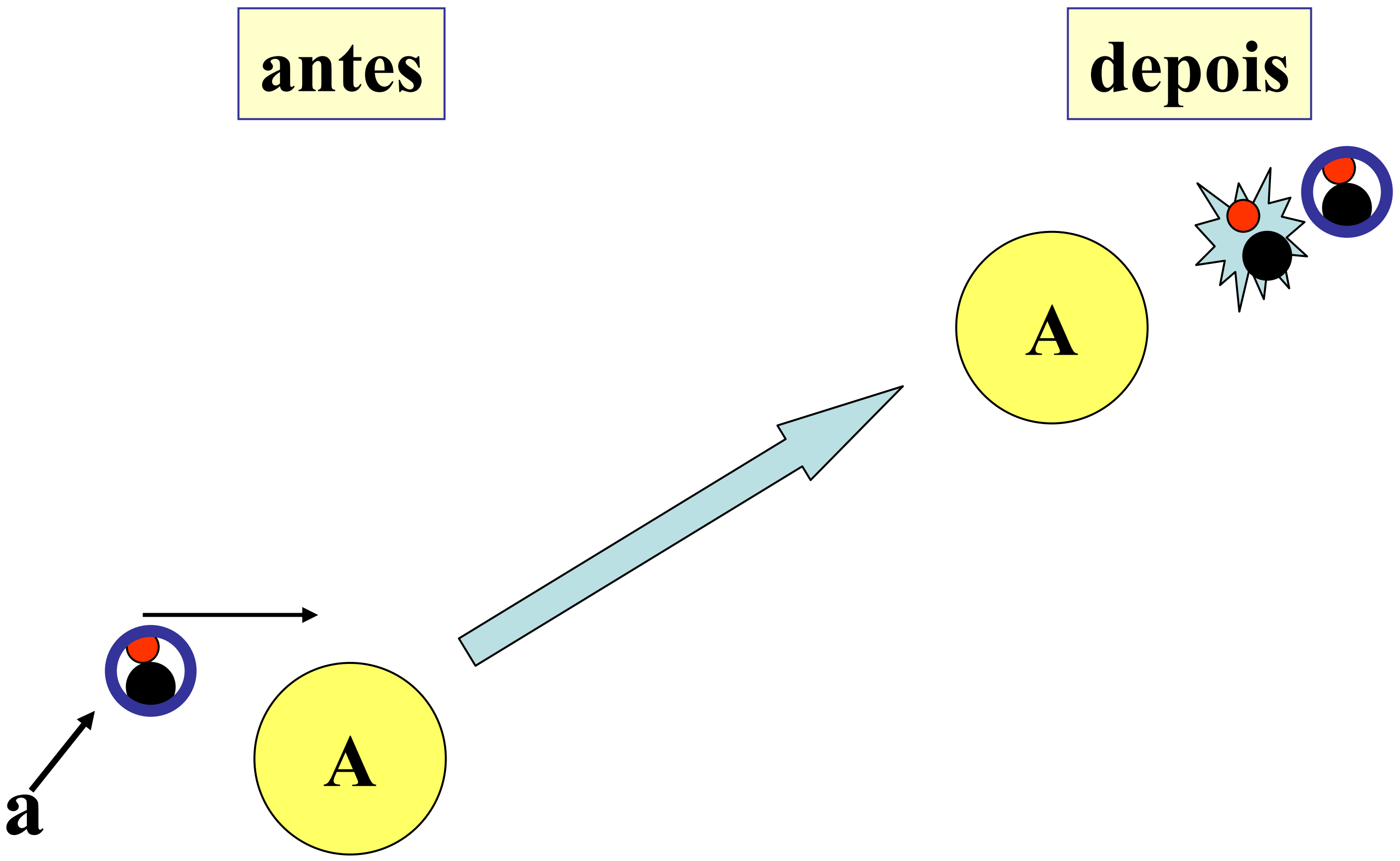
Note that $A+B+C+D=1$ and $AC=BD$

Note that $A+B+C+D=1$ and $AC=BD$

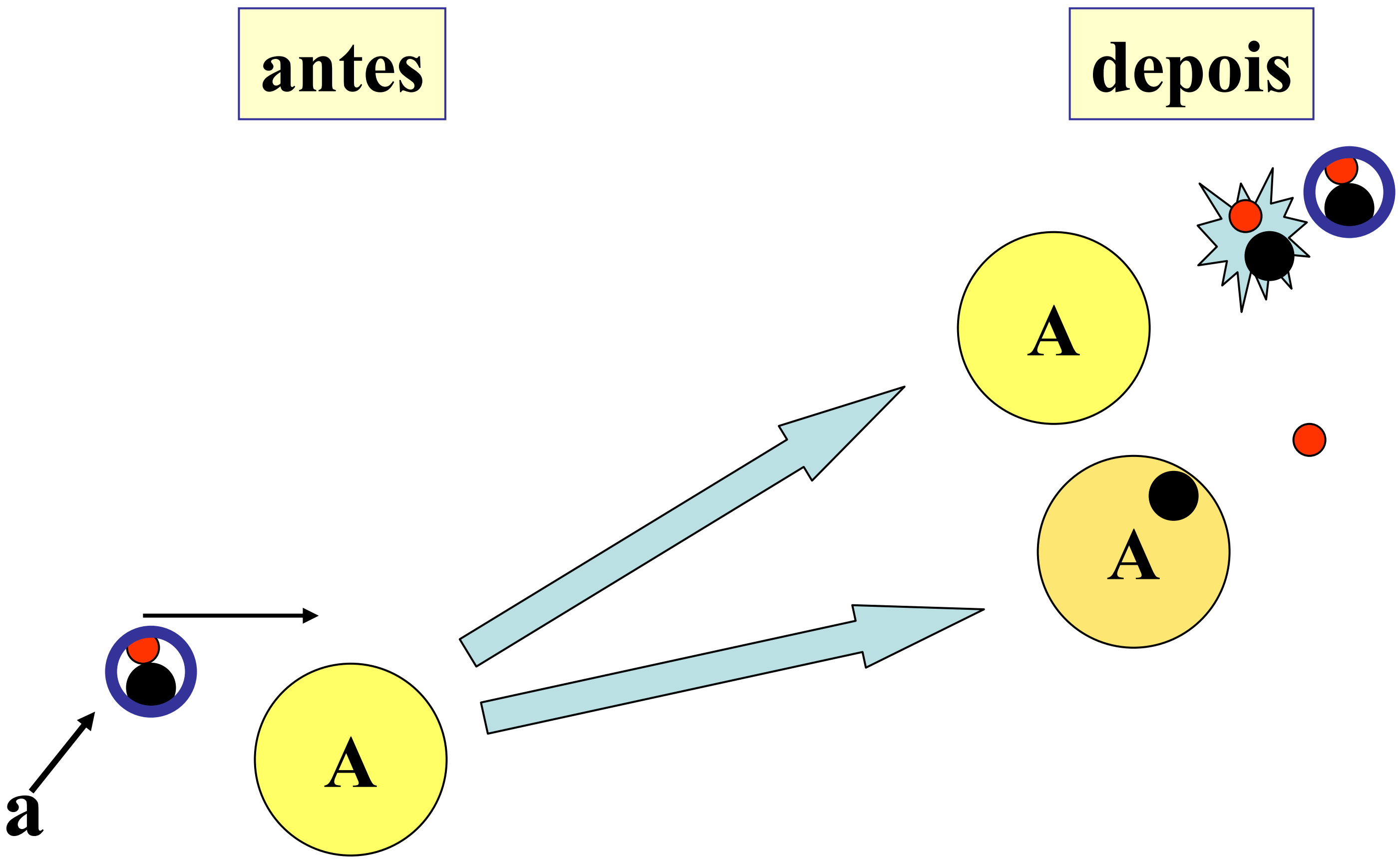
Lab energy of light product:	$\frac{E_3}{E_T}=B+D+2(AC)^{\frac{1}{2}}\cos\theta$ $=B[\cos\psi\pm(D/B-\sin^2\psi)^{\frac{1}{2}}]^2$		Use only plus sign unless $B>D$, in which case $\psi_{max}=\sin^{-1}(D/B)^{\frac{1}{2}}$
Lab energy of heavy product:	$\frac{E_4}{E_T}=A+C+2(AC)^{\frac{1}{2}}\cos\phi$ $=A[\cos\zeta\pm(C/A-\sin^2\zeta)^{\frac{1}{2}}]^2$		Use only plus sign unless $A>C$, in which case $\zeta_{max}=\sin^{-1}(C/A)^{\frac{1}{2}}$
Lab angle of heavy product:	$\sin\zeta=\left(\frac{M_3E_3}{M_4E_4}\right)^{\frac{1}{2}}\sin\psi$	C.M. angle of light product:	$\sin\theta=\left(\frac{E_3/E_T}{D}\right)\sin\psi$
Intensity or solid-angle ratio for light product:	$\frac{\sigma(\theta)}{\sigma(\psi)}=\frac{I(\theta)}{I(\psi)}=\frac{\sin\psi\,d\psi}{\sin\theta\,d\theta}=\frac{\sin^2\psi}{\sin^2\theta}\cos(\theta-\psi)=\frac{(AC)^{\frac{1}{2}}(D/B-\sin^2\psi)^{\frac{1}{2}}}{E_3/E_T}$		
Intensity or solid-angle ratio for heavy product:	$\frac{\sigma(\phi)}{\sigma(\zeta)}=\frac{I(\phi)}{I(\zeta)}=\frac{\sin\zeta\,d\zeta}{\sin\phi\,d\phi}=\frac{\sin^2\zeta}{\sin^2\phi}\cos(\phi-\zeta)=\frac{(AC)^{\frac{1}{2}}(C/A-\sin^2\zeta)^{\frac{1}{2}}}{E_4/E_T}$		
Intensity or solid-angle ratio for associated particles in the lab system:	$\frac{\sigma(\zeta)}{\sigma(\psi)}=\frac{I(\zeta)}{I(\psi)}=\frac{\sin\psi\,d\psi}{\sin\zeta\,d\zeta}=\frac{\sin^2\psi\cos(\theta-\psi)}{\sin^2\zeta\cos(\phi-\zeta)}$		



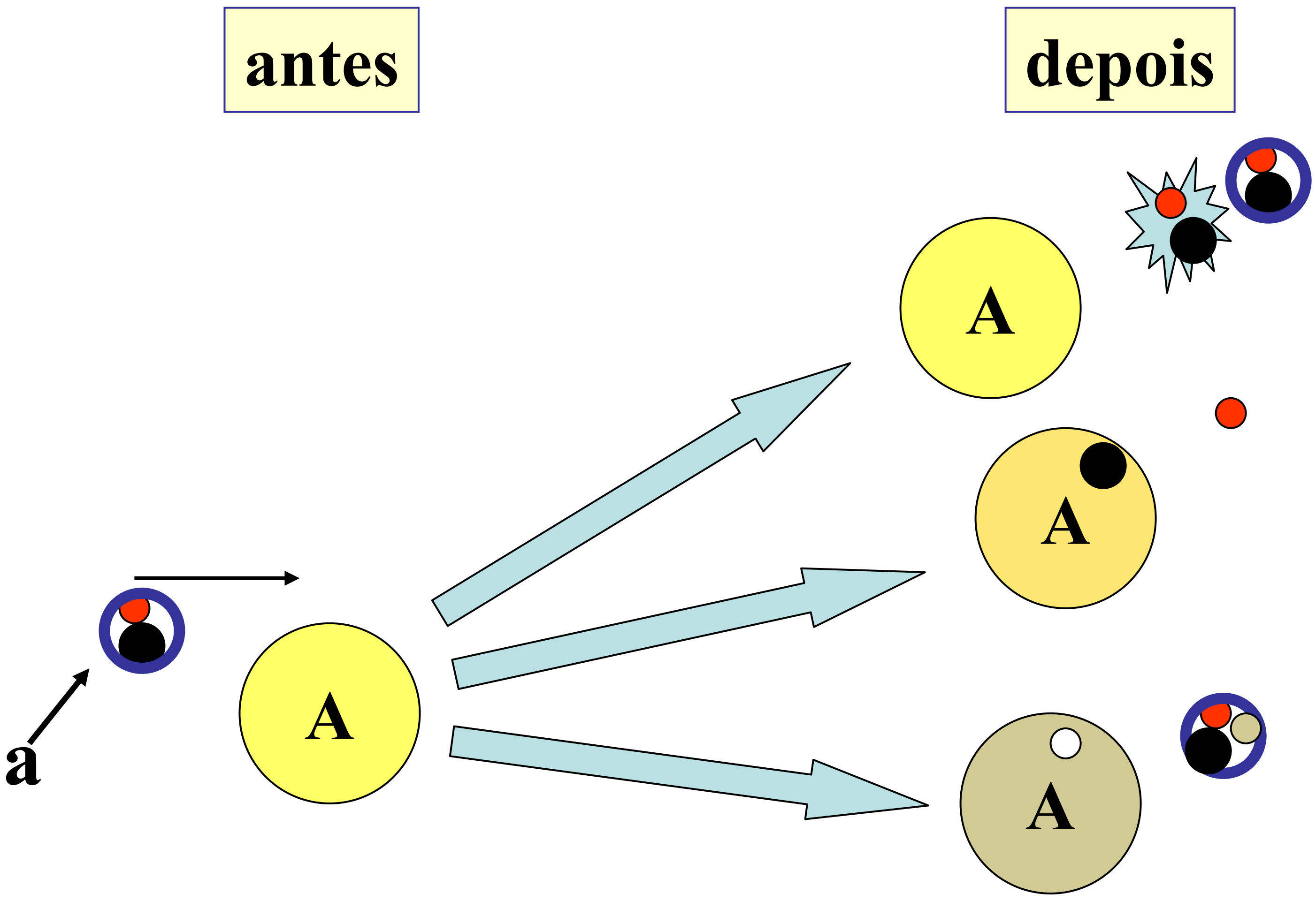
E_OUT (MeV)
E_RES (MeV)
120
100
80
60
40
20
0
0 20 40 60 80 100 120 140 160 180
θ_LAB
E_LAB = 100 MeV
208Pb(16O,16O)
40Ca(16O,16O)
28Si(16O,16O)
20Ne(16O,16O)
16O(16O,16O)
12C(16O,16O)
6Li(16O,16O)



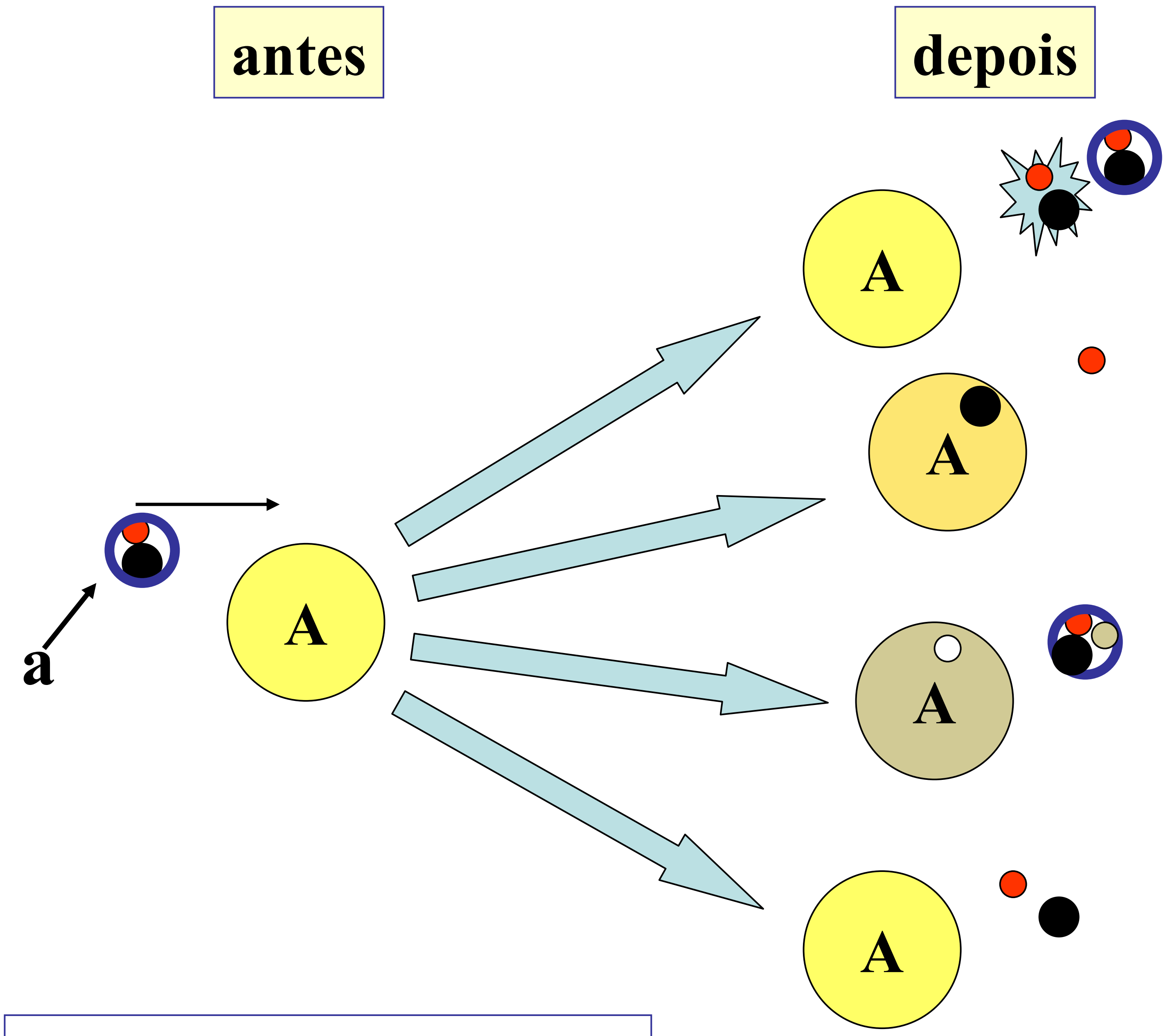
Reações diretas (rápidas)



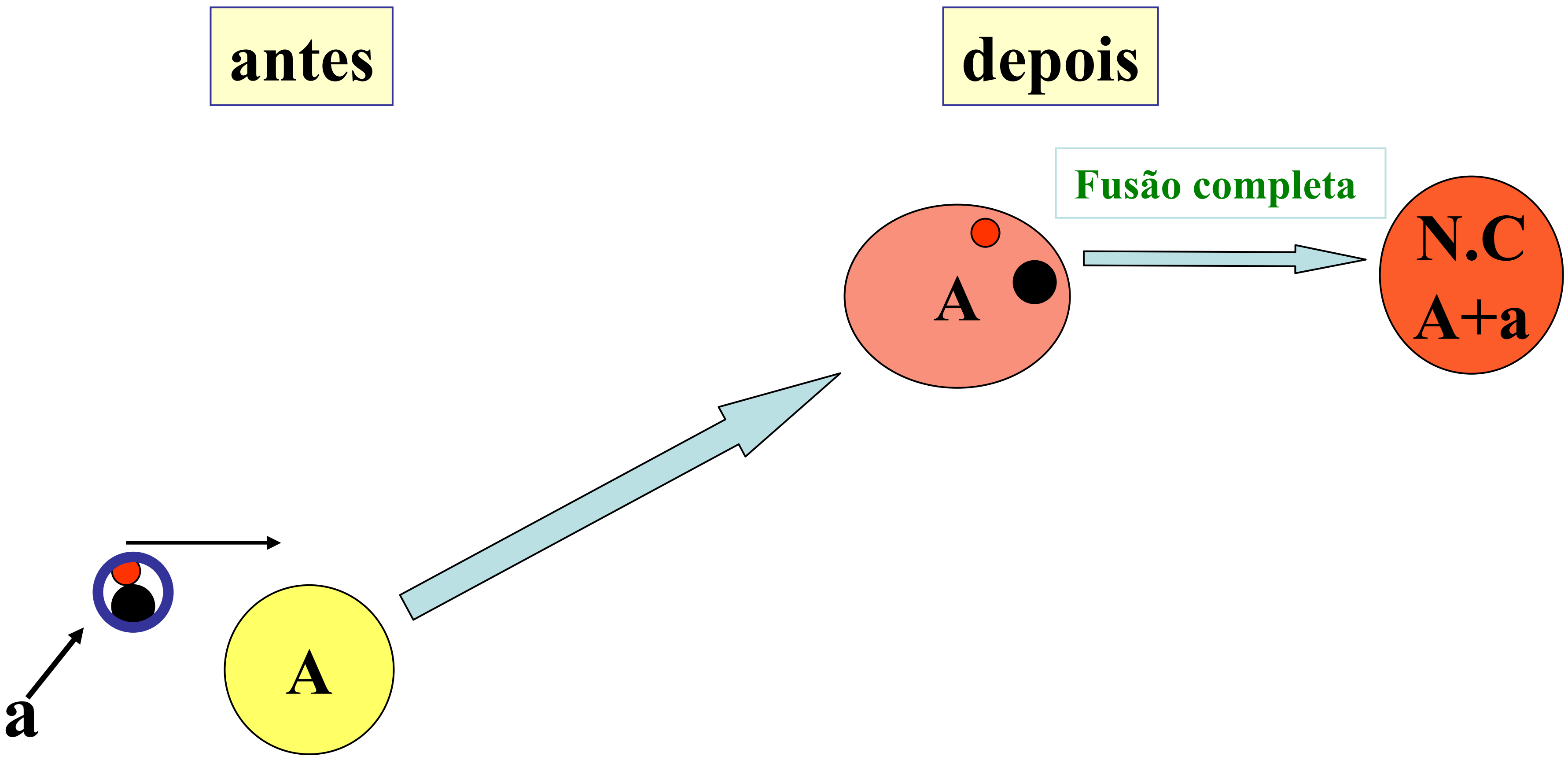
Reações diretas (rapidas)



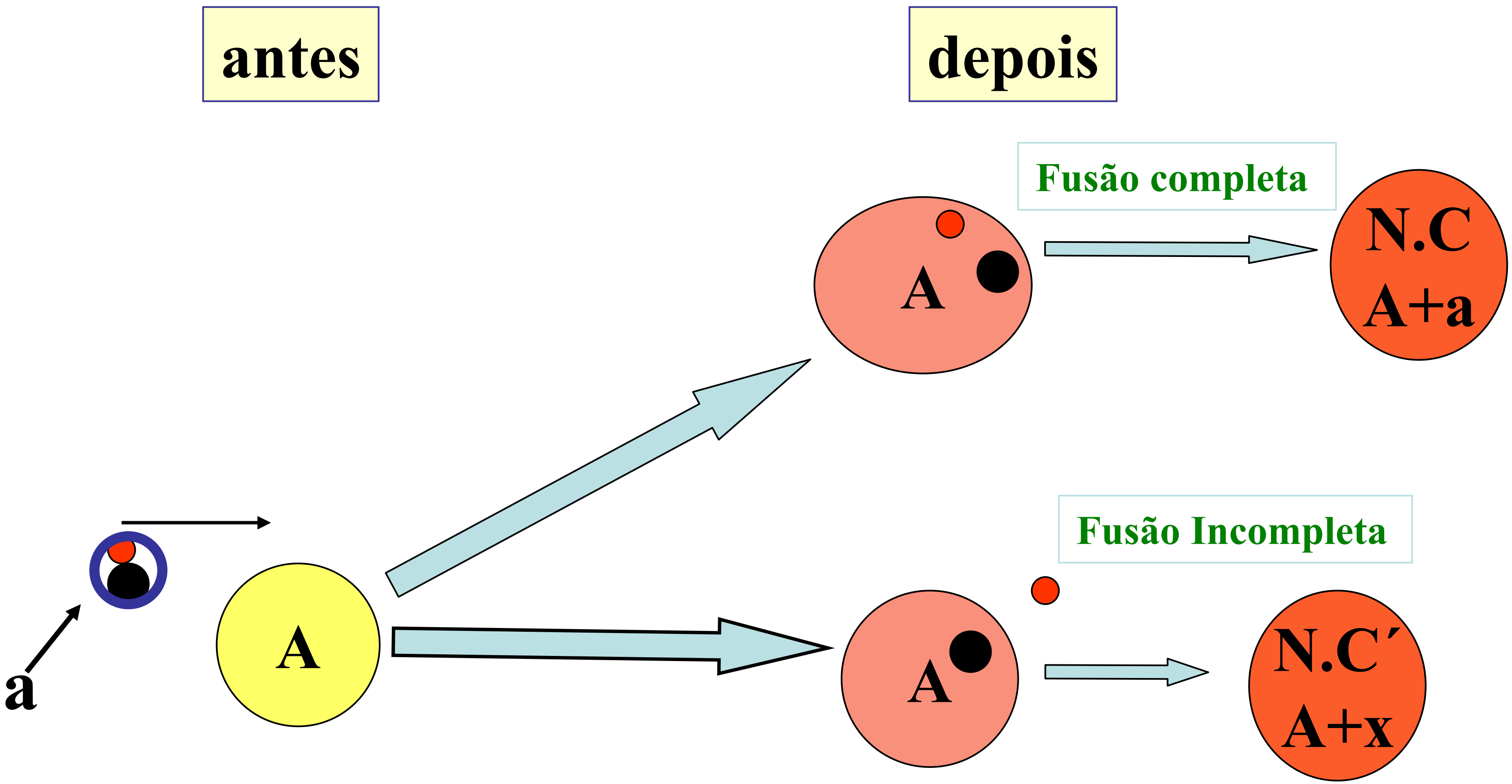
Reações diretas (rapidas)



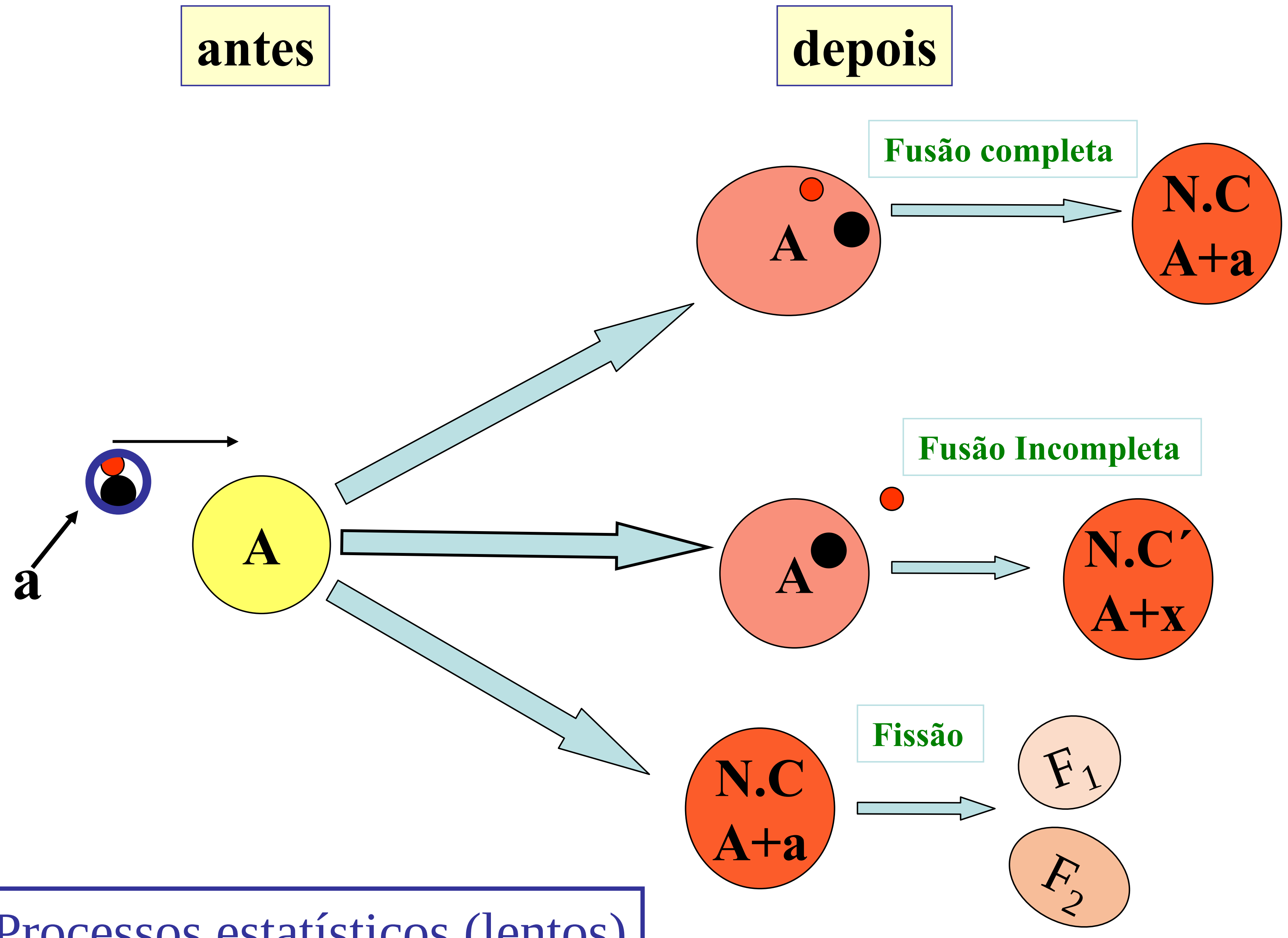
Reações diretas (rapidas)



Processos estatísticos (lentos) | via **Núcleo Composto (N.C.)**



Processos estatísticos (lentos)



Processos estatísticos (lentos)

ESPALHAMENTO ELASTICO

REAÇÕES DIRETAS

ESPALHAMENTO INELASTICO

TRANSFERENCIA DE NUCLEONS
- **"STRIPPING"**
- **"PICK-UP"**

"KNOCK-OUT"

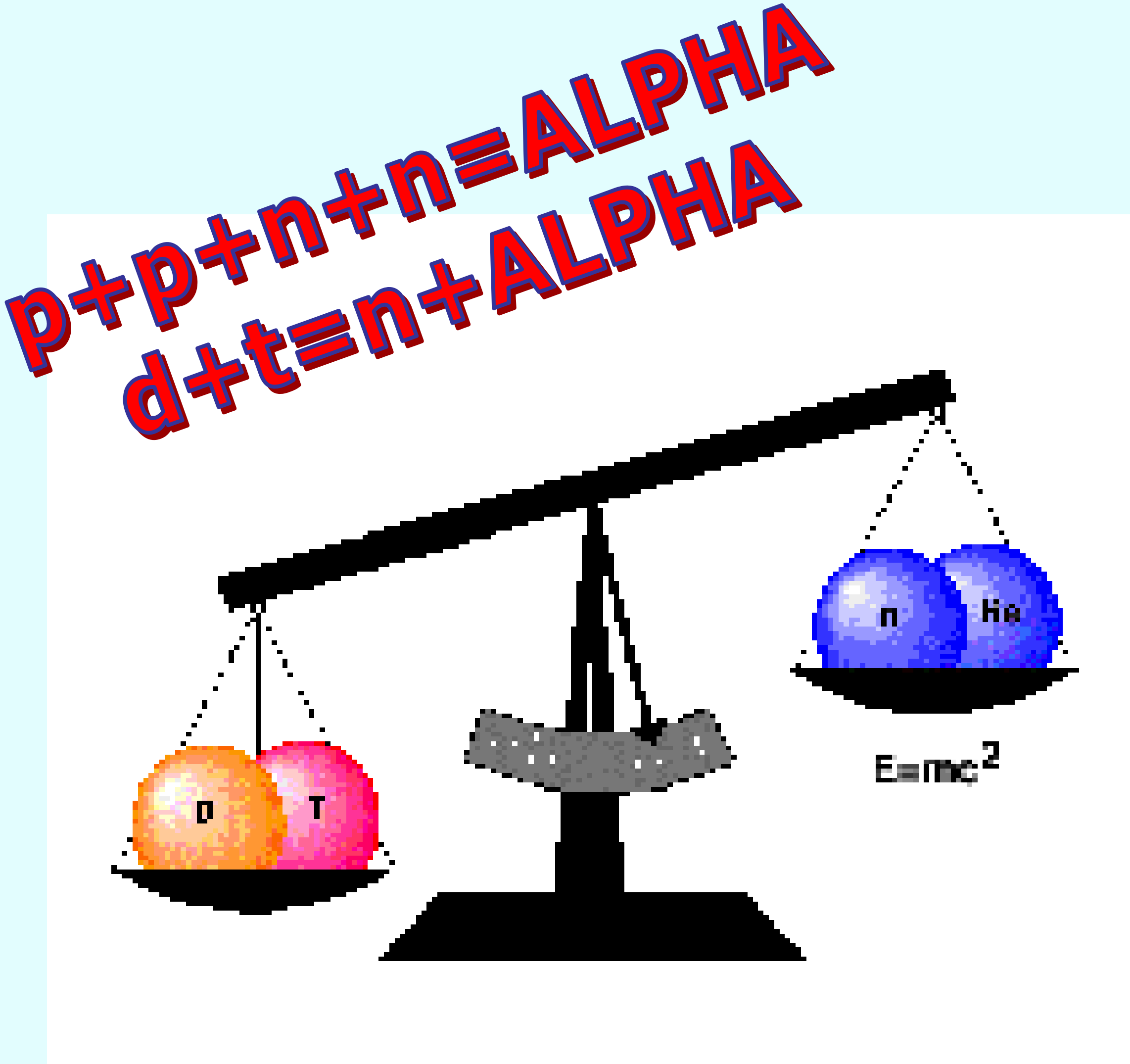
QUEBRA NUCLEAR ("BREAK-UP")

PRÉ-EQUILIBRIO

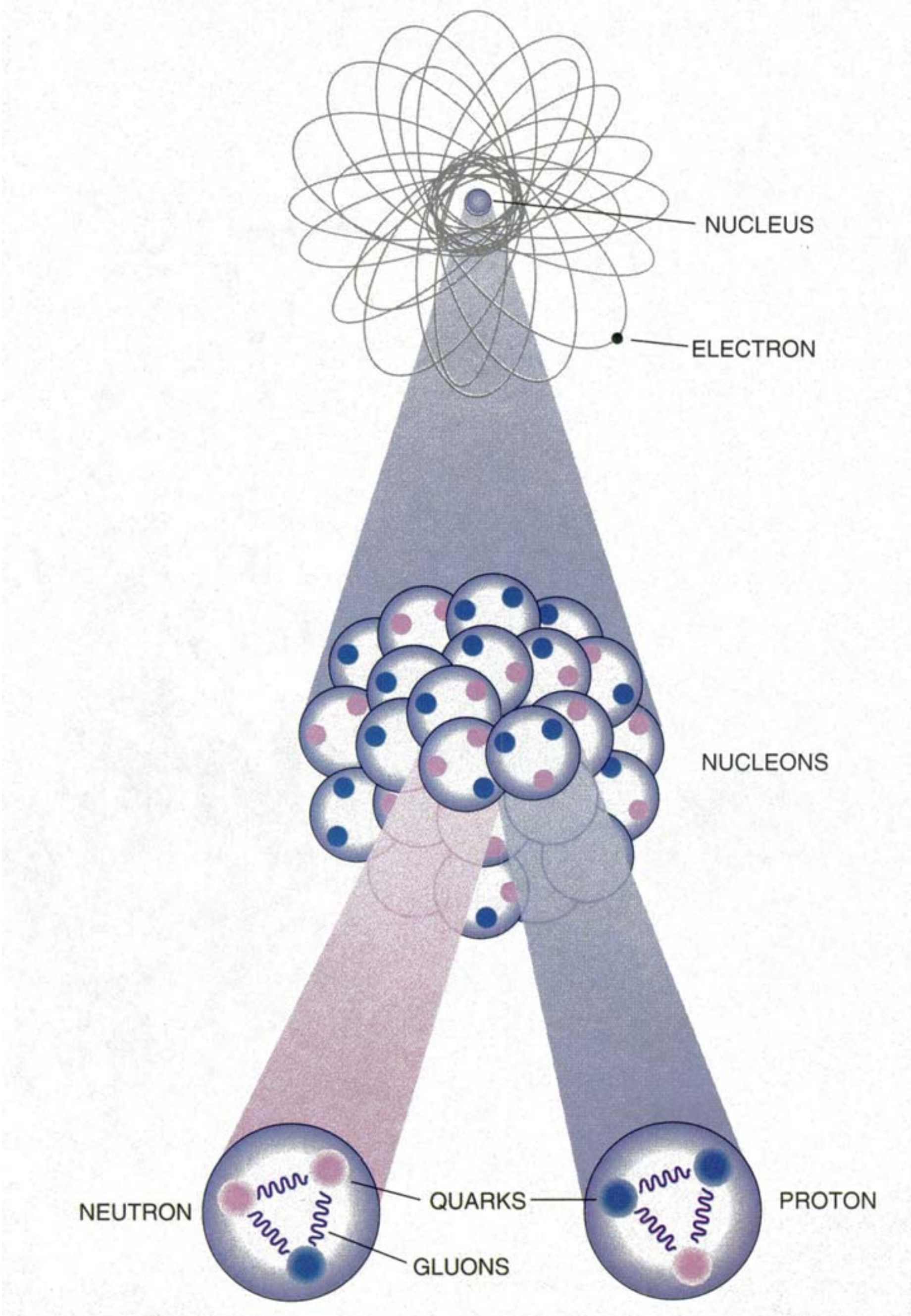
NUCLEO COMPOSTO

FUSÃO
- **FUSÃO COMPLETA**
- **FUSÃO INCOMPLETA**

FISSÃO



n	=1.00866 u.m.a.
p	= 1.0079 u.m.a.
d	= 2.01410 u.m.a.
t	= 3.01860 u.m.a.
^{4}He	= 4. 00260 u.m.a.
^{6}Li	= 6. 01512 u.m.a.
^{12}C	= 0.00000 u.m.a.



$M_{(proton)} = M_p = 938.27$ MeV

$M_{(neutron)} = M_n = 939.56$ MeV

$M_{(eletron)} = M_e = 0.511$ MeV

$M_{(\text{átomo de Hidrogênio})} = M_H = 938.58$ MeV

$M_p + M_e = 938.27 + 0.511 = 938.78$ MeV

$M_H = 938.58$ MeV

$M_{(16O)} = 14\,899.17$ MeV

$8M_p + 8M_n = 15\,022.64$ MeV

$\Delta M = 123.47$ MeV

p = (u + u + d)

$M_{(u)} \approx 3$ MeV

$M_{(d)} \approx 6$ MeV

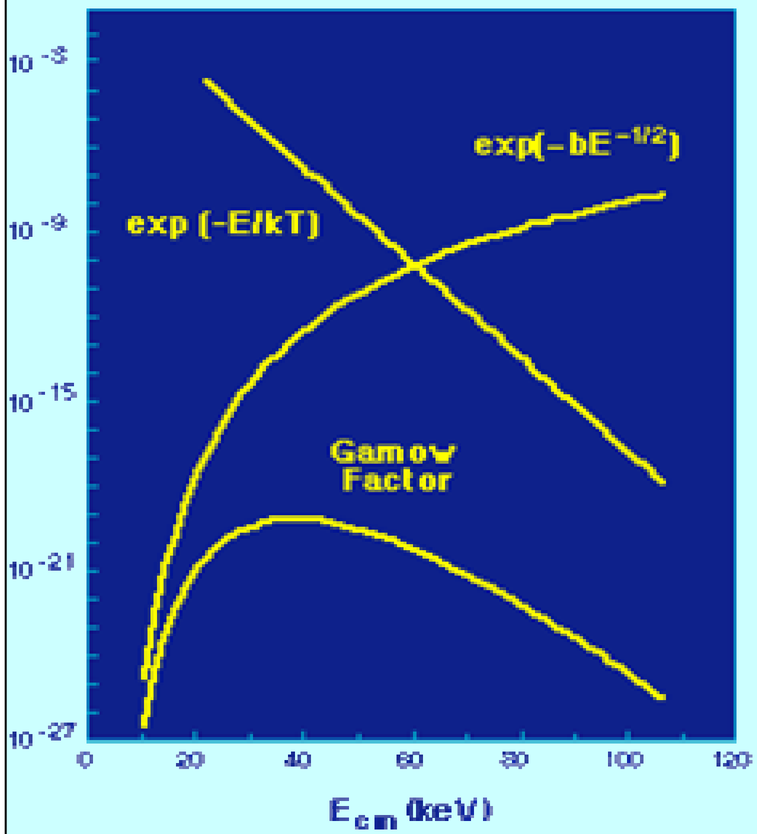
$2\,M(u) + M(d) \approx 12$ MeV

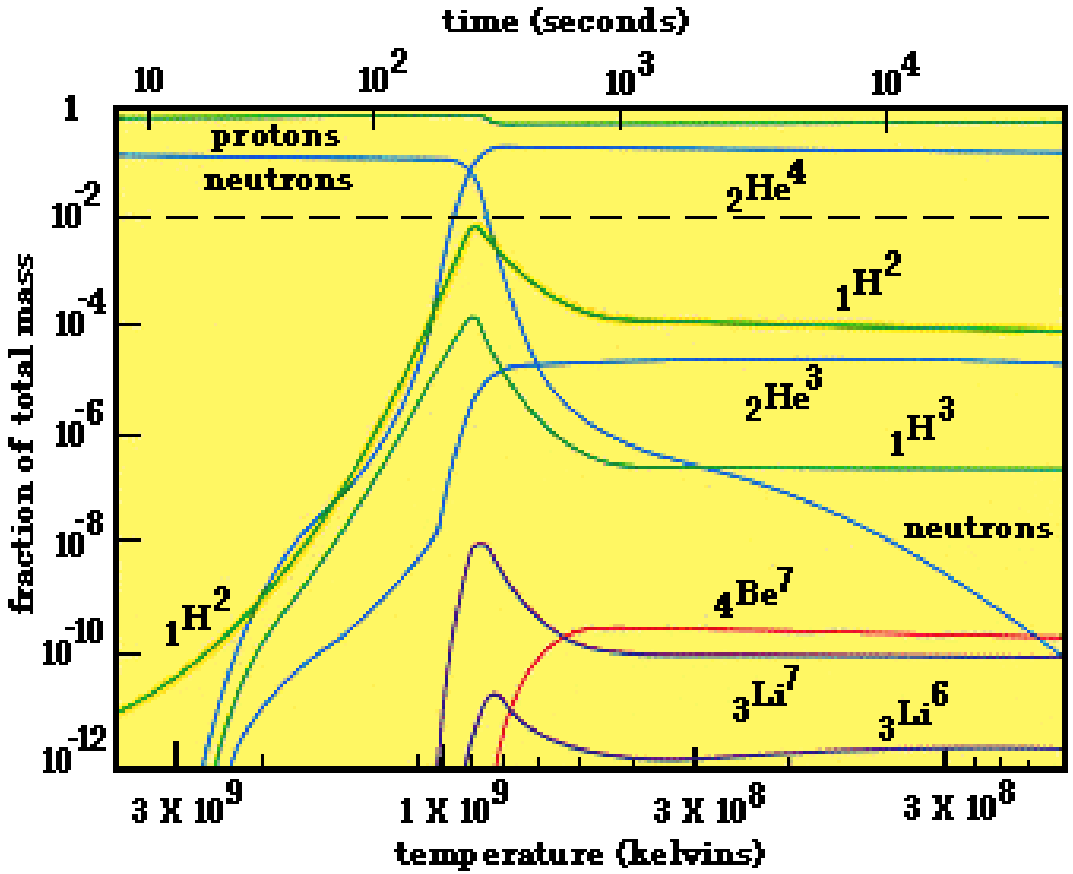
$Mp = 938.27$ MeV

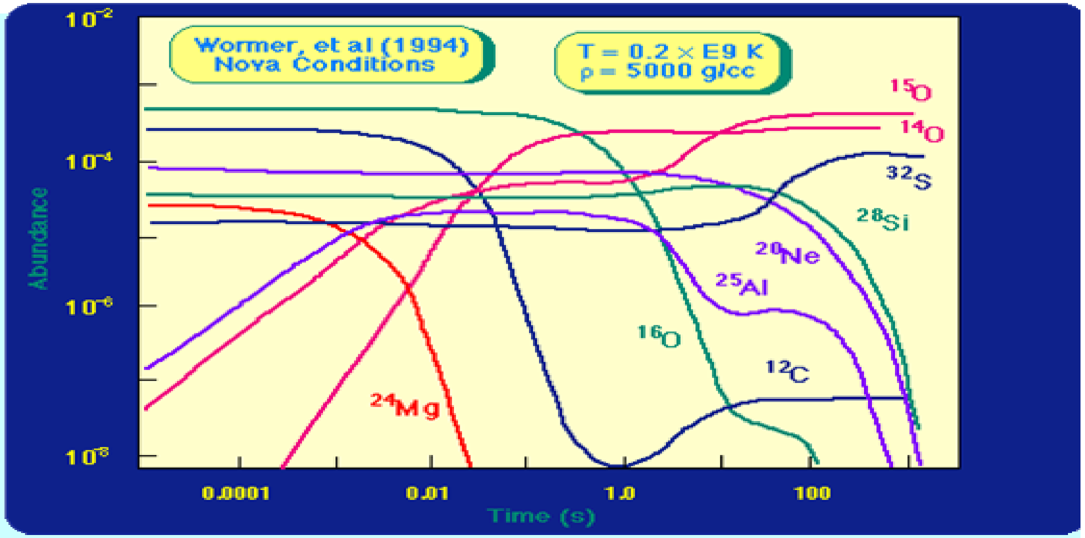
$\Delta M \approx 926$ MeV

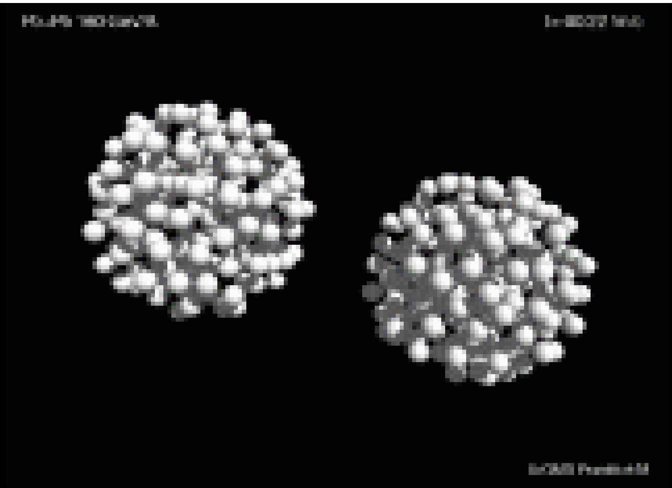
ESPALHAMENTO

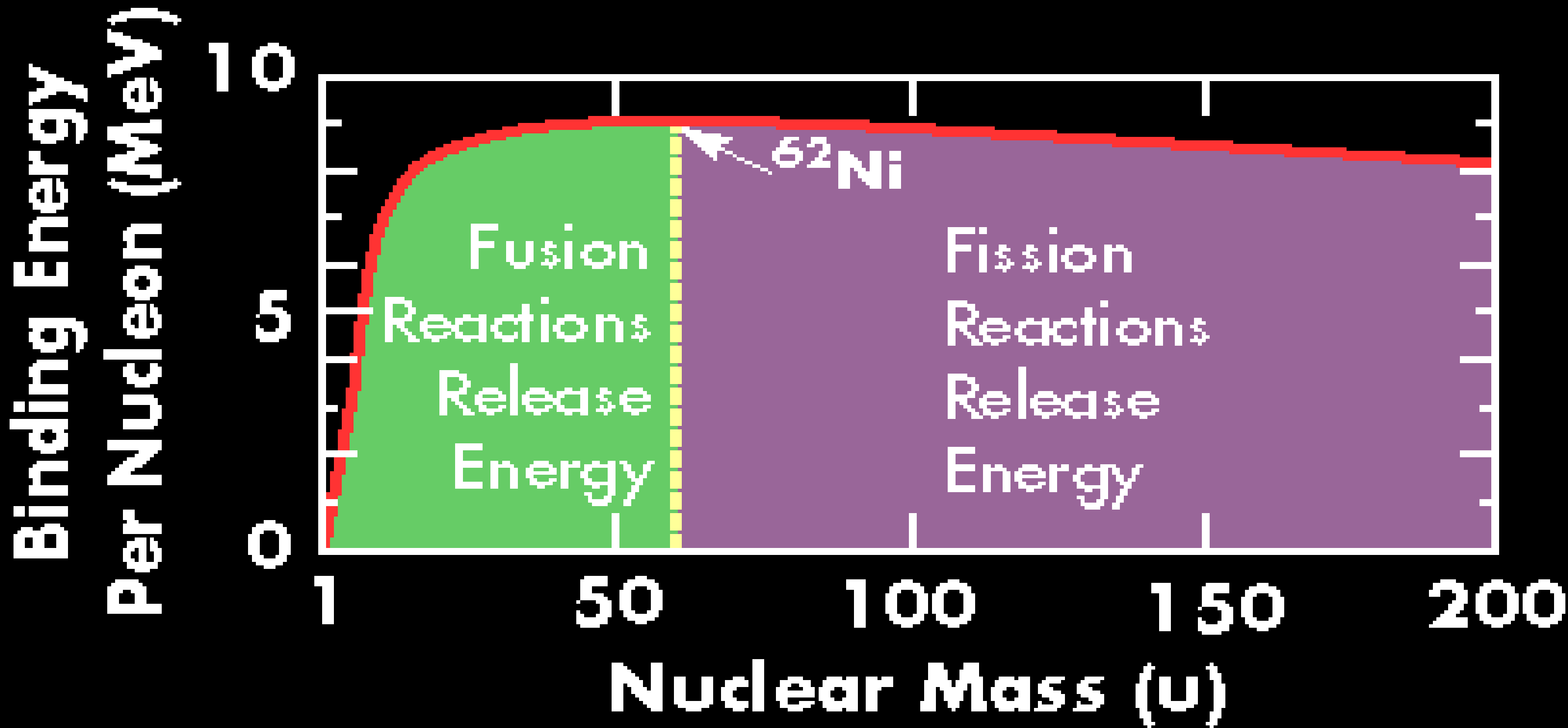
TUNELAMENTO


exp(-bE^-1/2)
exp (-E/kT)
Gamow Factor
E cm (keV)
0
20
40
60
80
100
120


time (seconds)
10
10^2
10^3
10^4
protons
neutrons
2He4
1H2
2He3
1H3
neutrons
4Be7
3Li7
3Li6
fraction of total mass
3 X 10^9
1 X 10^9
3 X 10^8
3 X 10^8
temperature (kelvins)


Wormer, et al (1994)
Nova Conditions
T = 0.2 × E9 K
ρ = 5000 g/cc
15O
14O
32S
28Si
20Ne
25Al
16O
12C
24Mg
Abundance
0.0001
0.01
1.0
100
Time (s)





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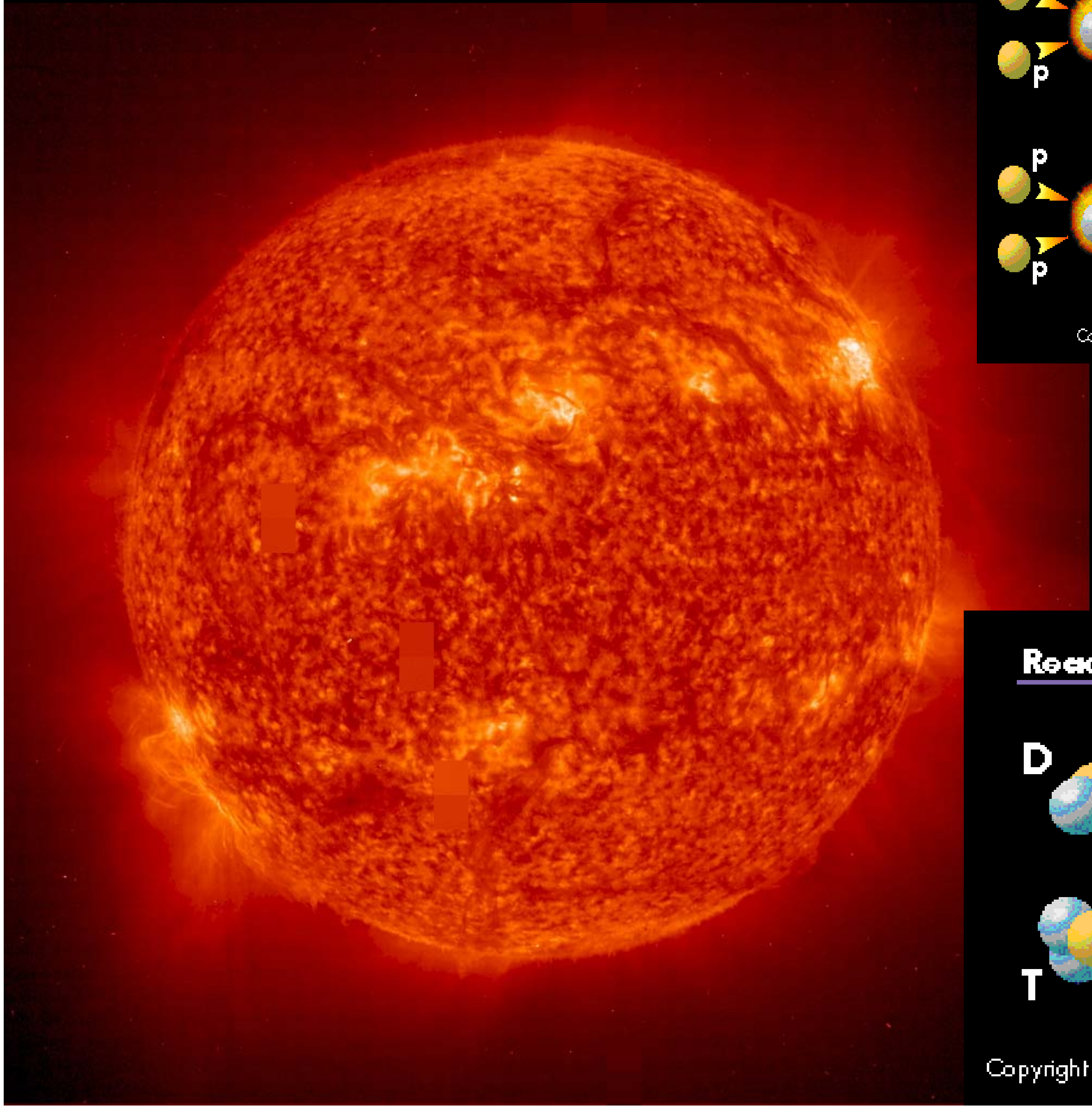
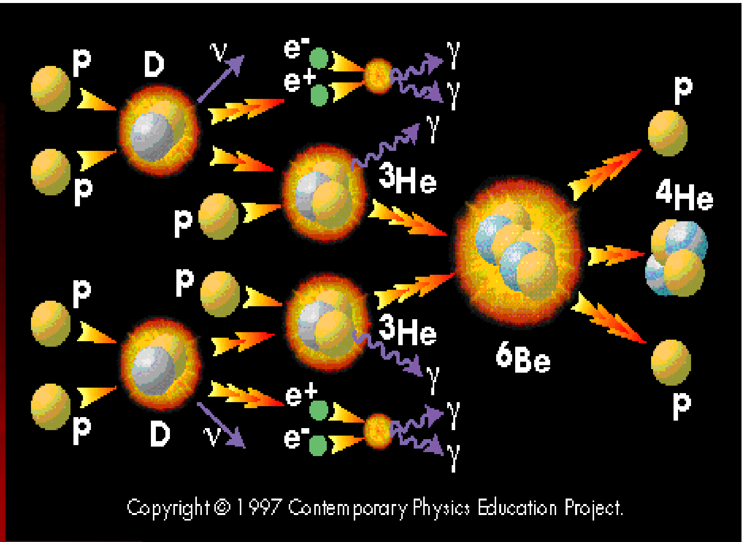
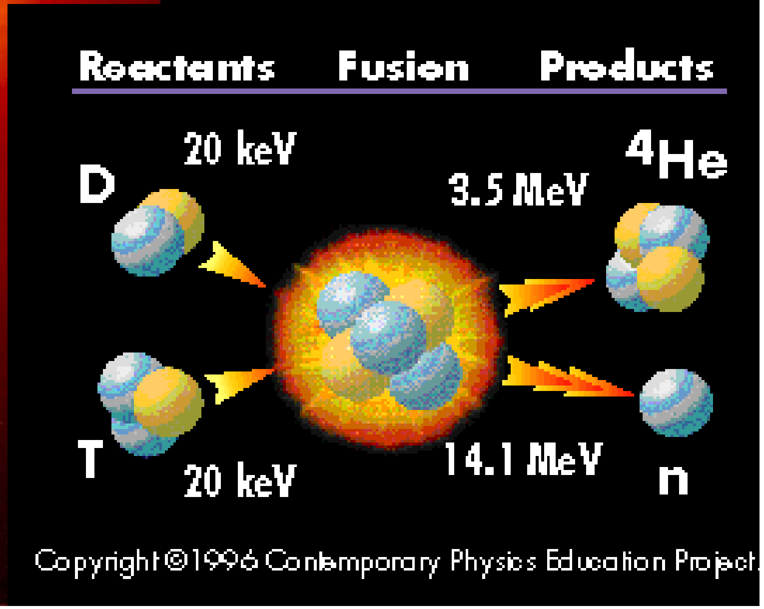
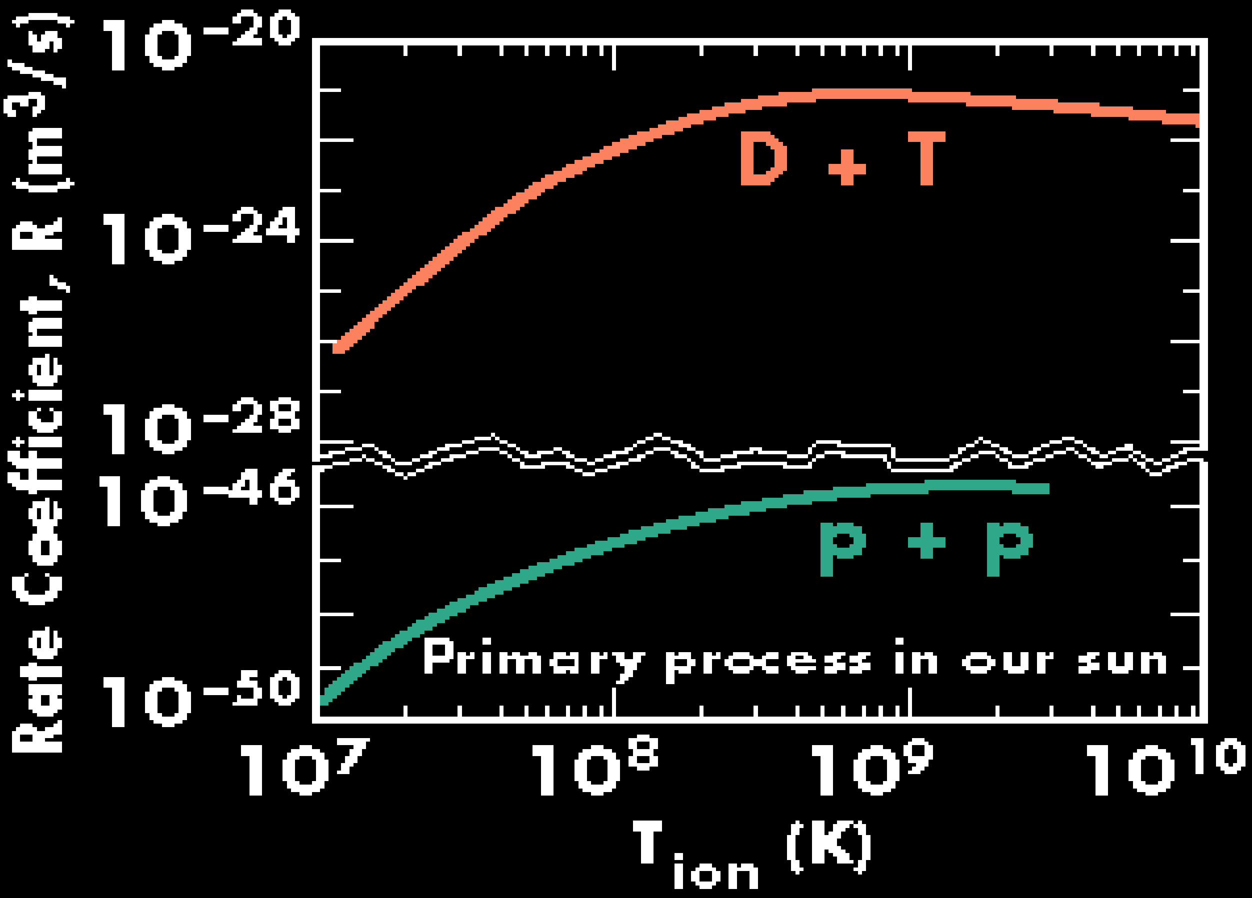



p
p
D
ν
e-
e+
γ
γ
γ
3He
p
p
p
3He
γ
D
ν
e+
e-
γ
γ
6Be
p
4He
p
Copyright © 1997 Contemporary Physics Education Project.


Reactants
Fusion
Products
D
20 keV
T
20 keV
3.5 MeV
4He
14.1 MeV
n
Copyright ©1996 Contemporary Physics Education Project.



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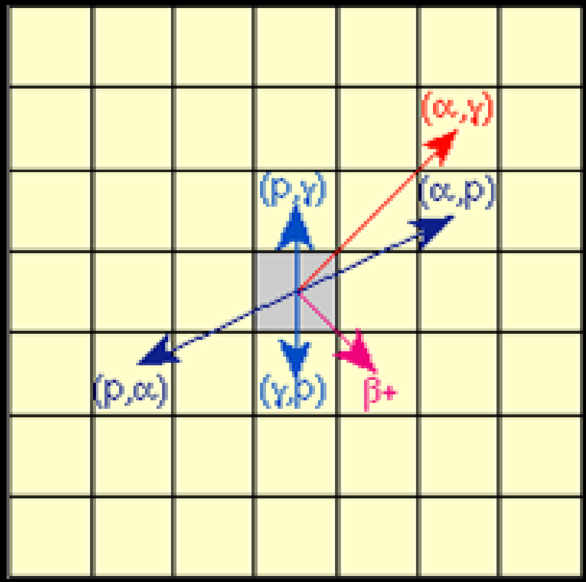
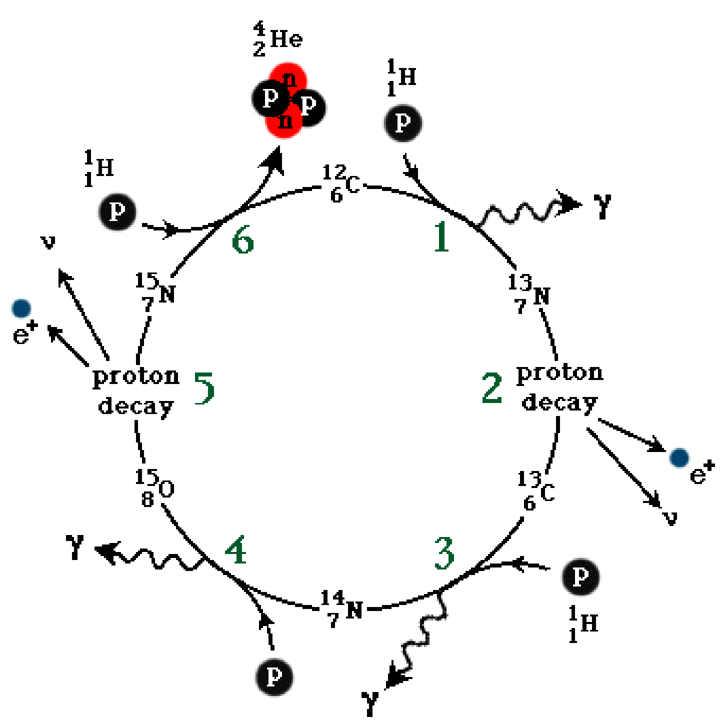
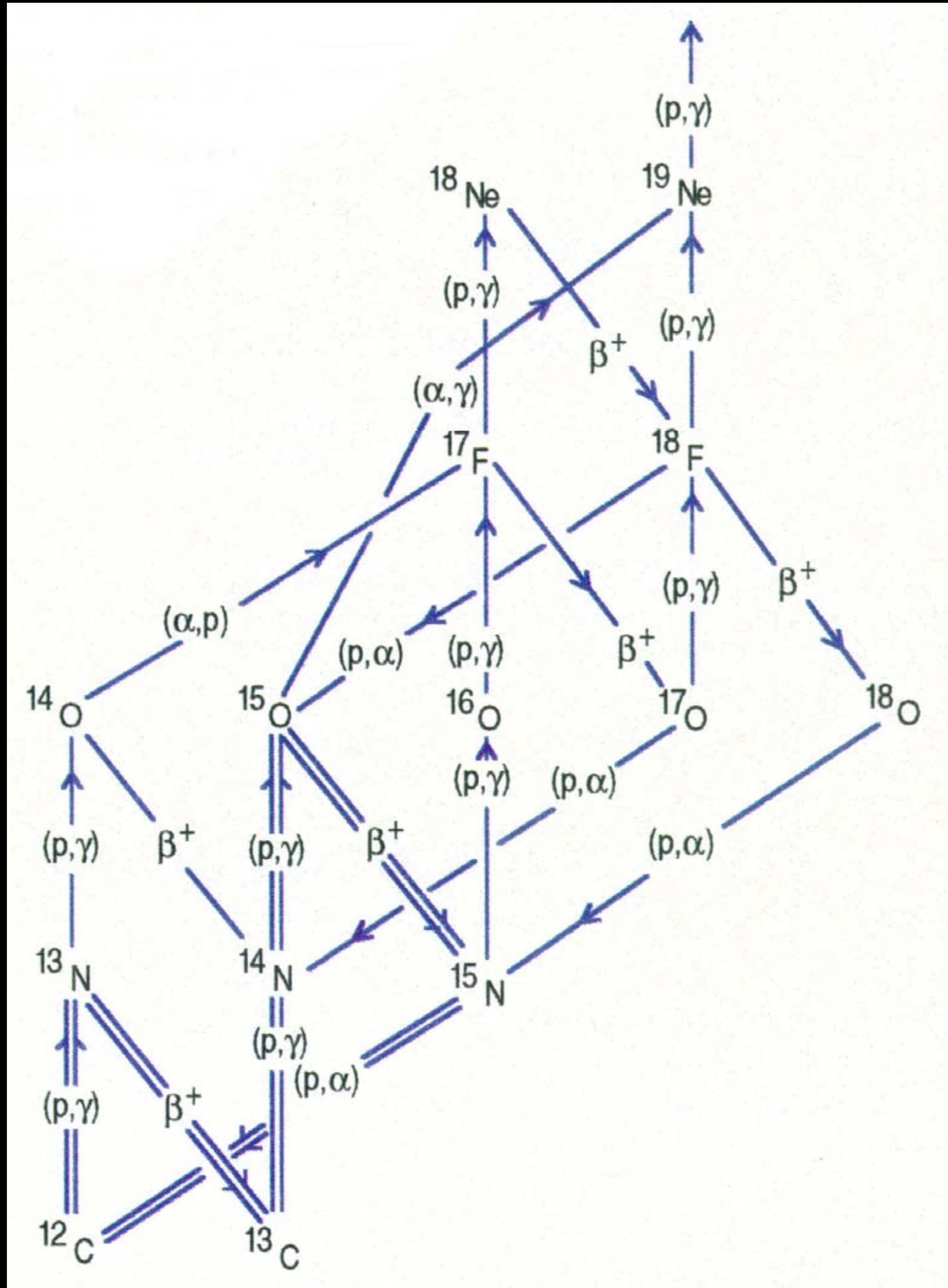
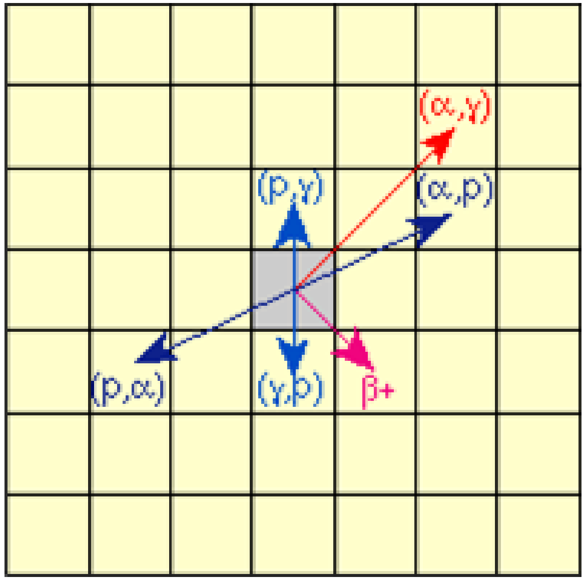


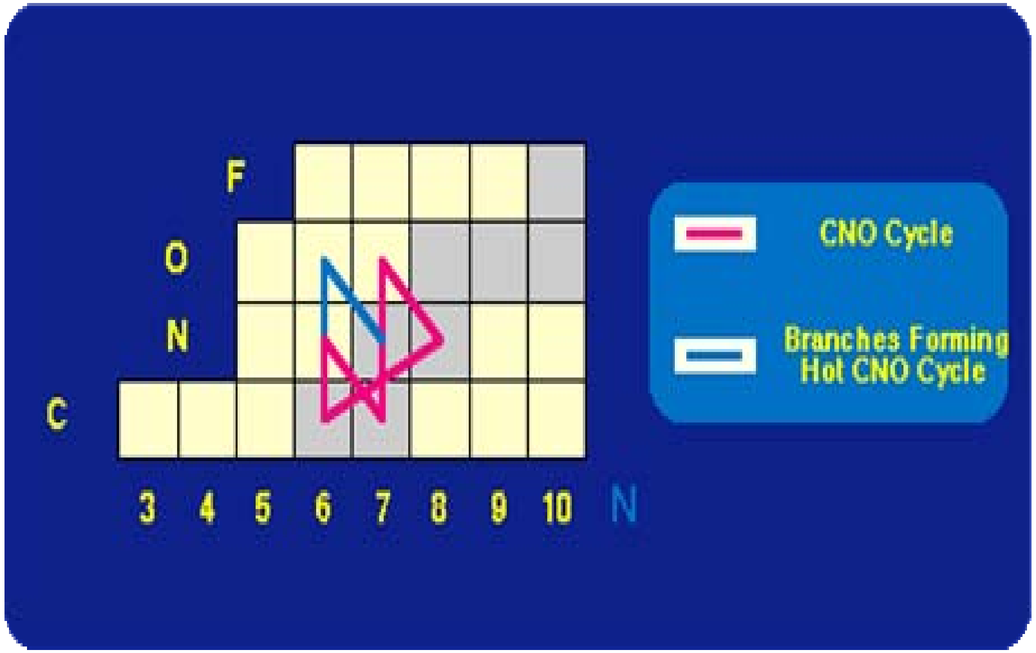
Carbon-Nitrogen-Oxygen Cycle

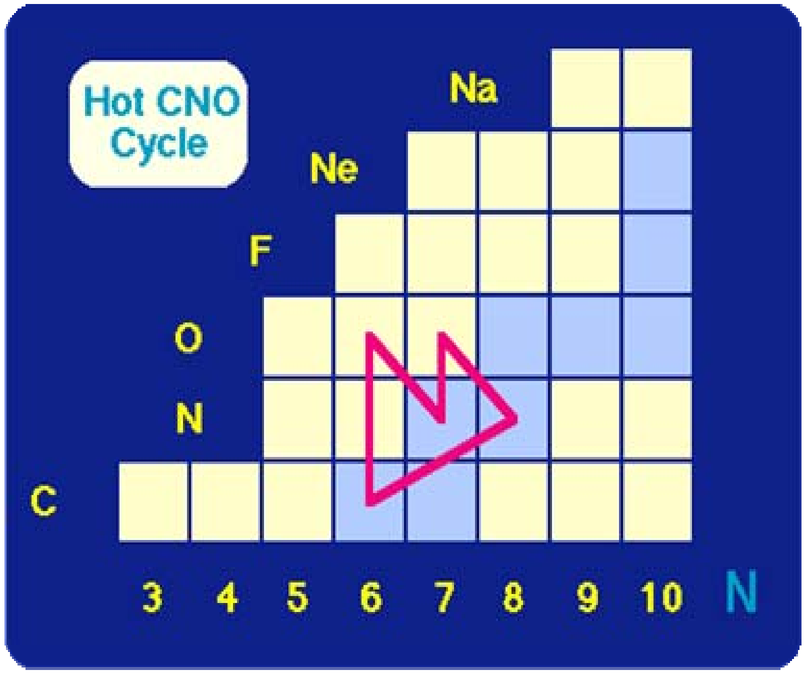


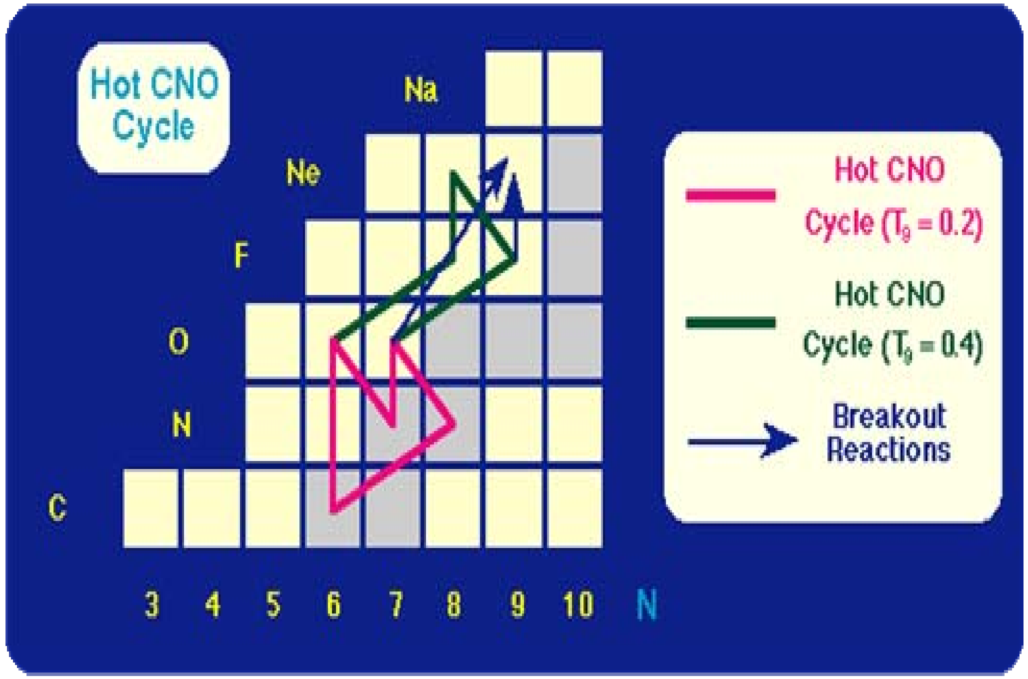


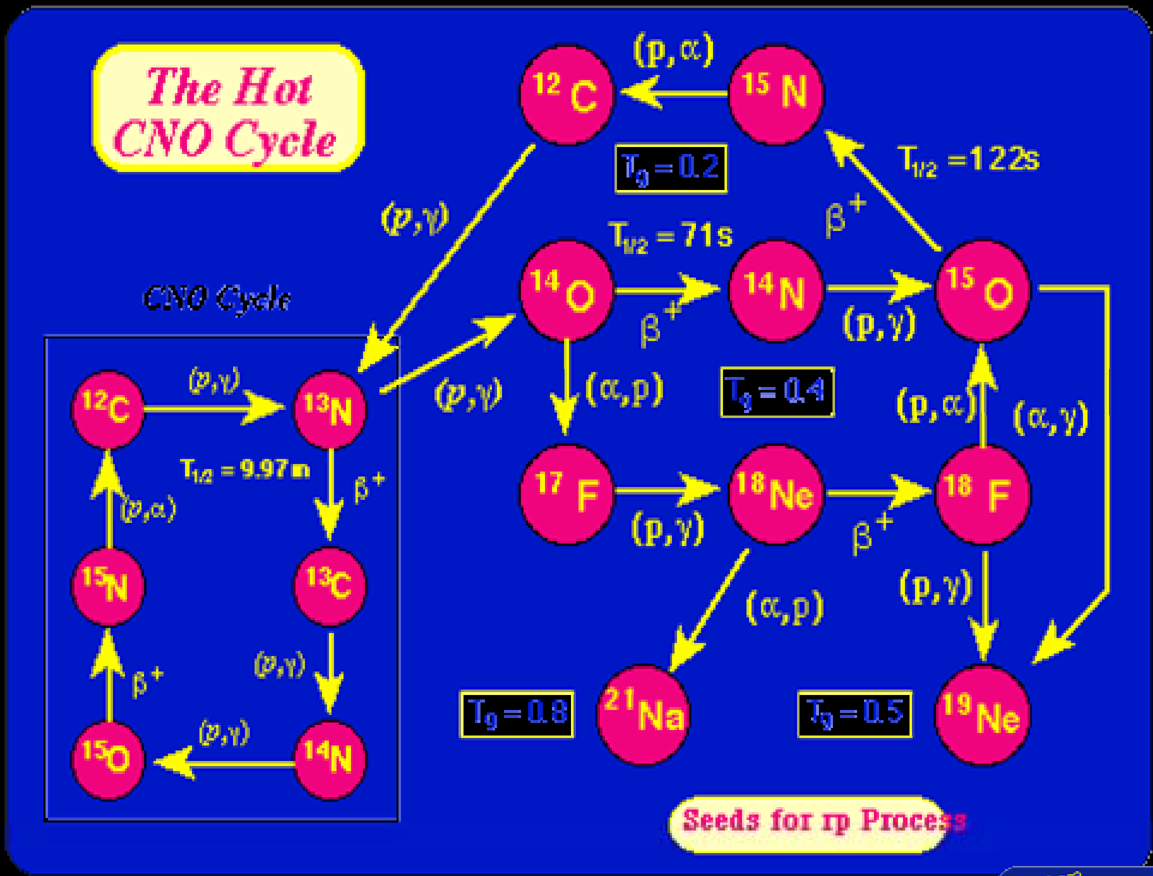


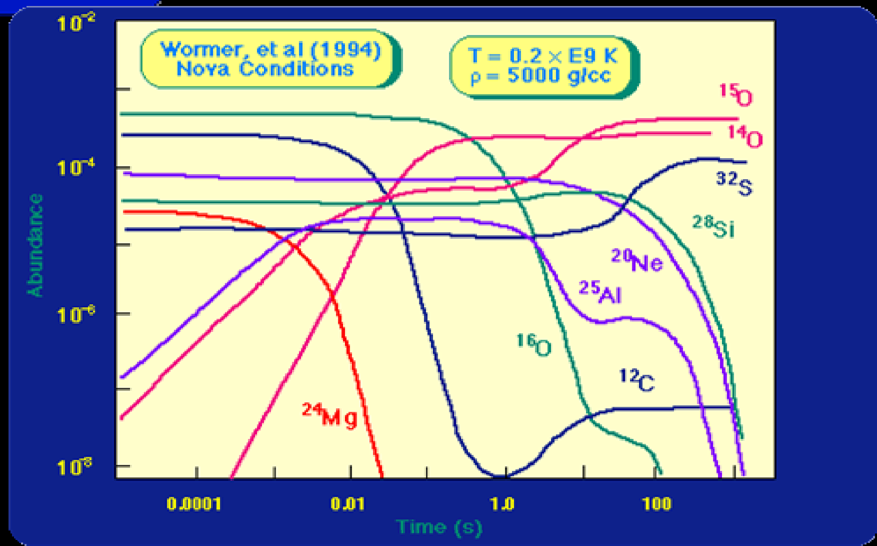
Reaction Vectors

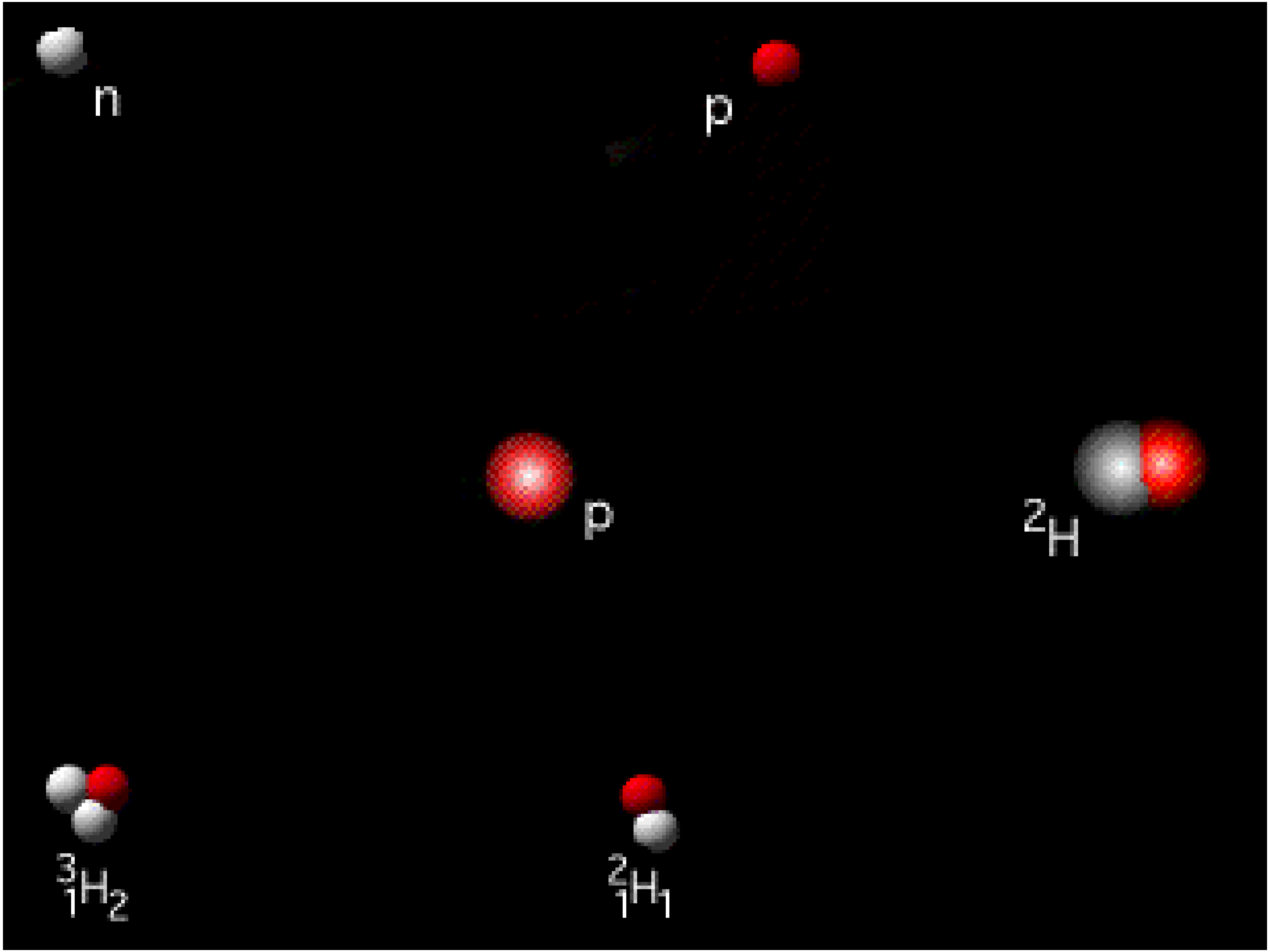


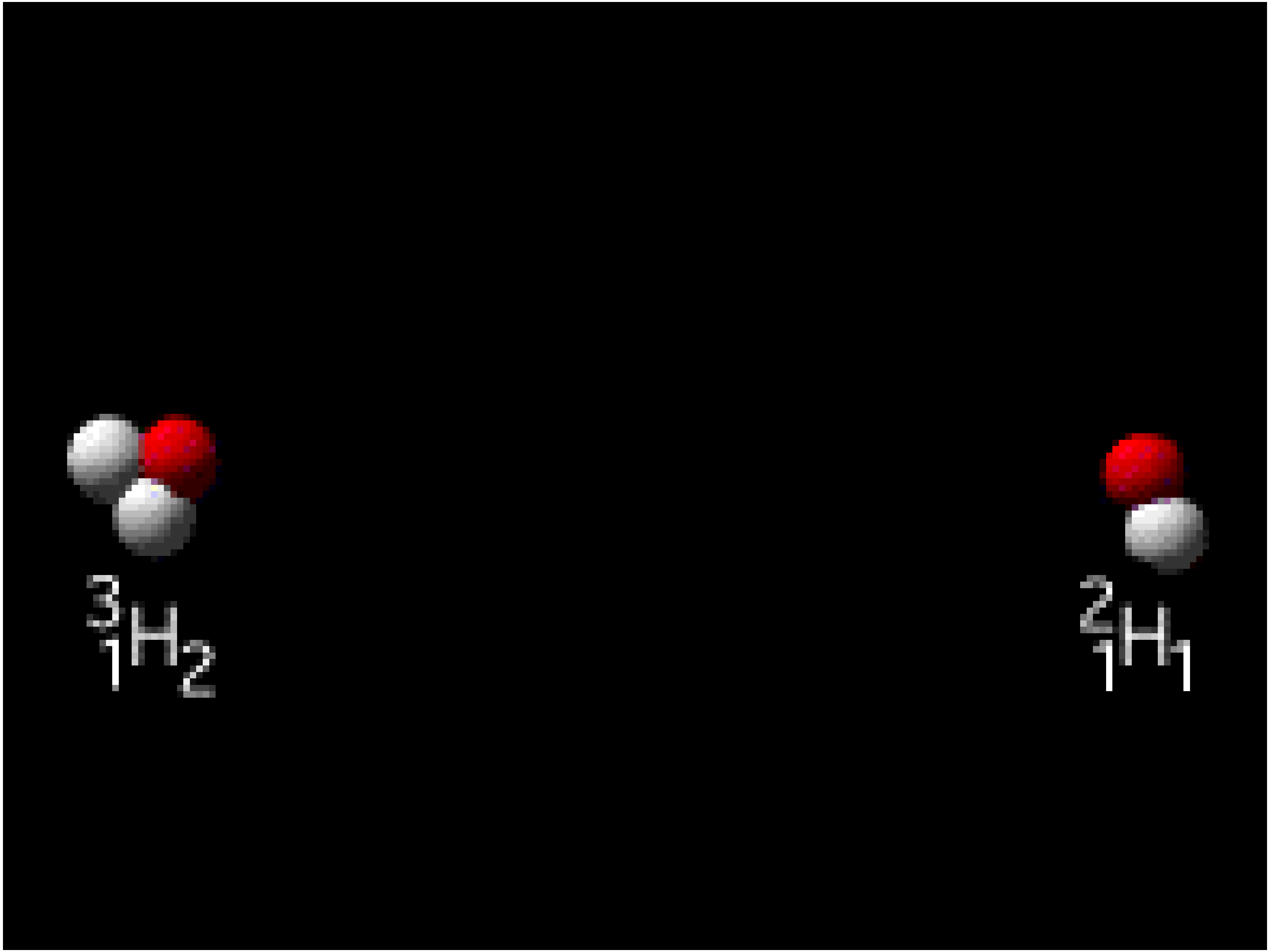


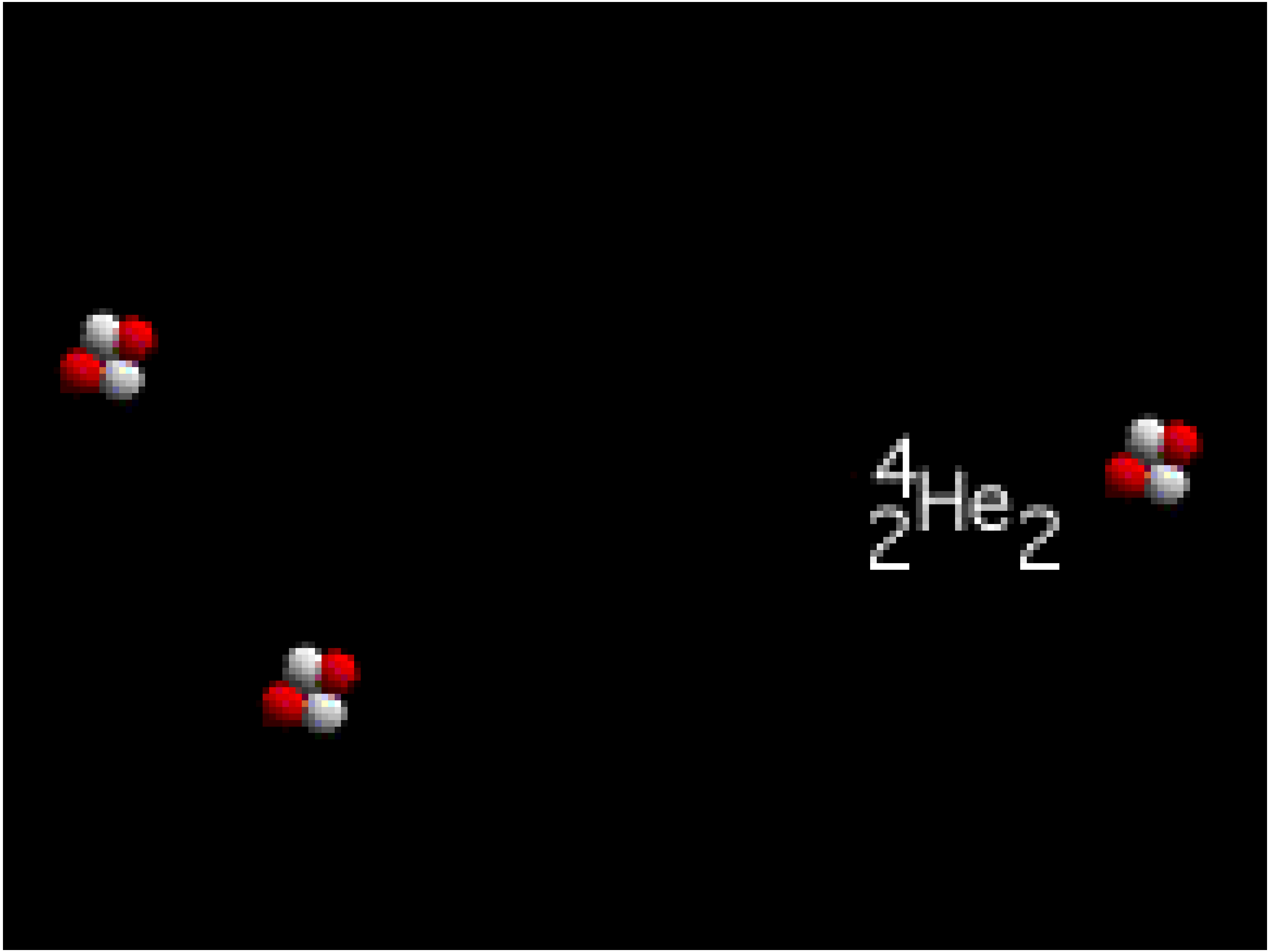


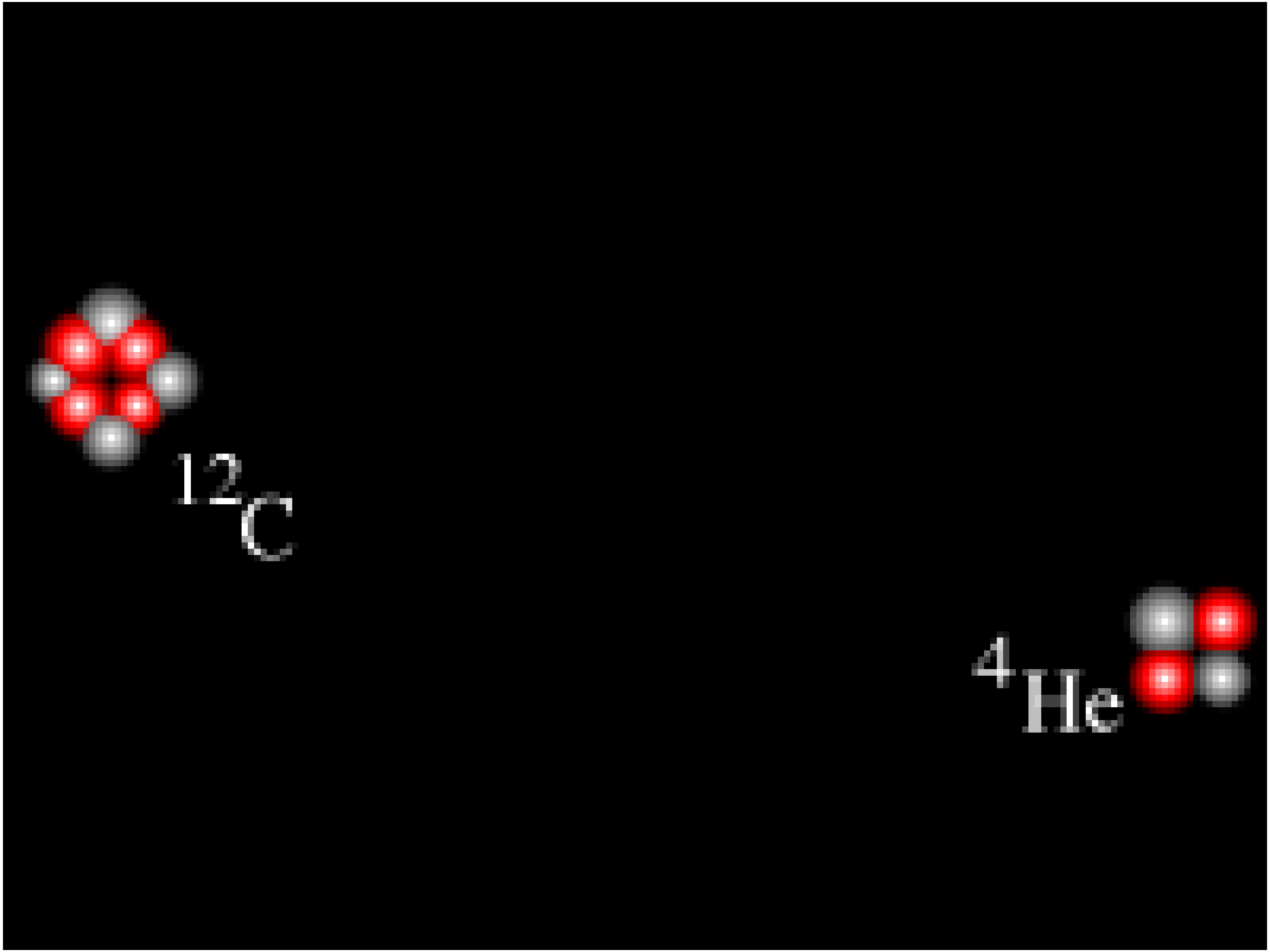

The Hot CNO Cycle
CNO Cycle
12C
13N
13C
14N
15O
15N
14O
15N
17F
18Ne
18F
19Ne
21Na
(p,γ)
(p,α)
β+
T1/2 = 9.97 m
T1/2 = 71s
T1/2 = 122s
(α,p)
(α,γ)
T9 = 0.2
T9 = 0.4
T9 = 0.8
T9 = 0.5
Seeds for rp Process


Wormer, et al (1994)
Nova Conditions
T = 0.2 × E9 K
ρ = 5000 g/cc
Abundance
Time (s)
0.0001
0.01
1.0
100
15O
14O
32S
28Si
20Ne
25Al
16O
12C
24Mg

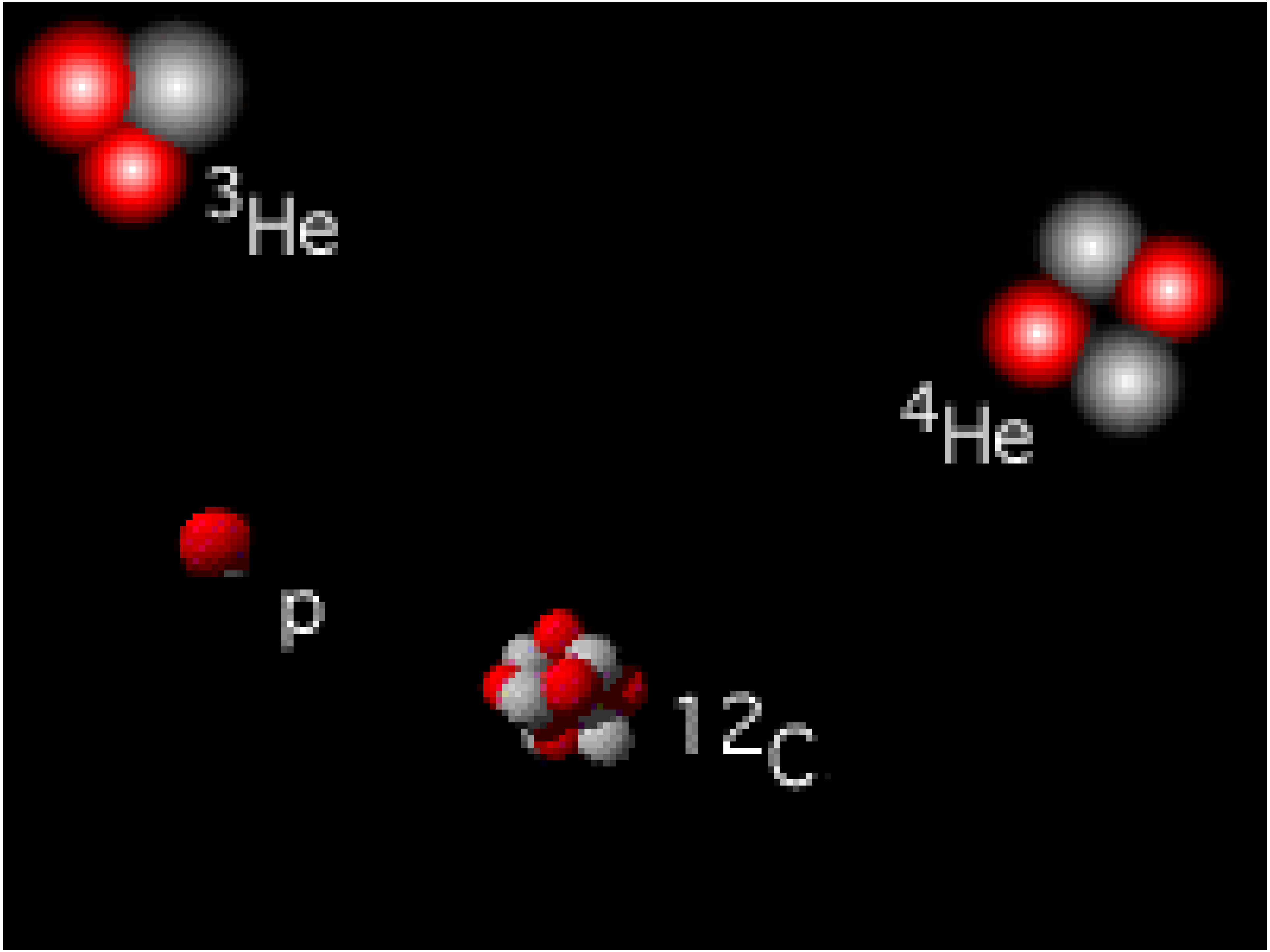

n
p
p
^{2}H
$^{3}_{1}H_{2}$
$^{2}_{1}H_{1}$


$^{3}_{1}H_2$
$^{2}_{1}H_1$

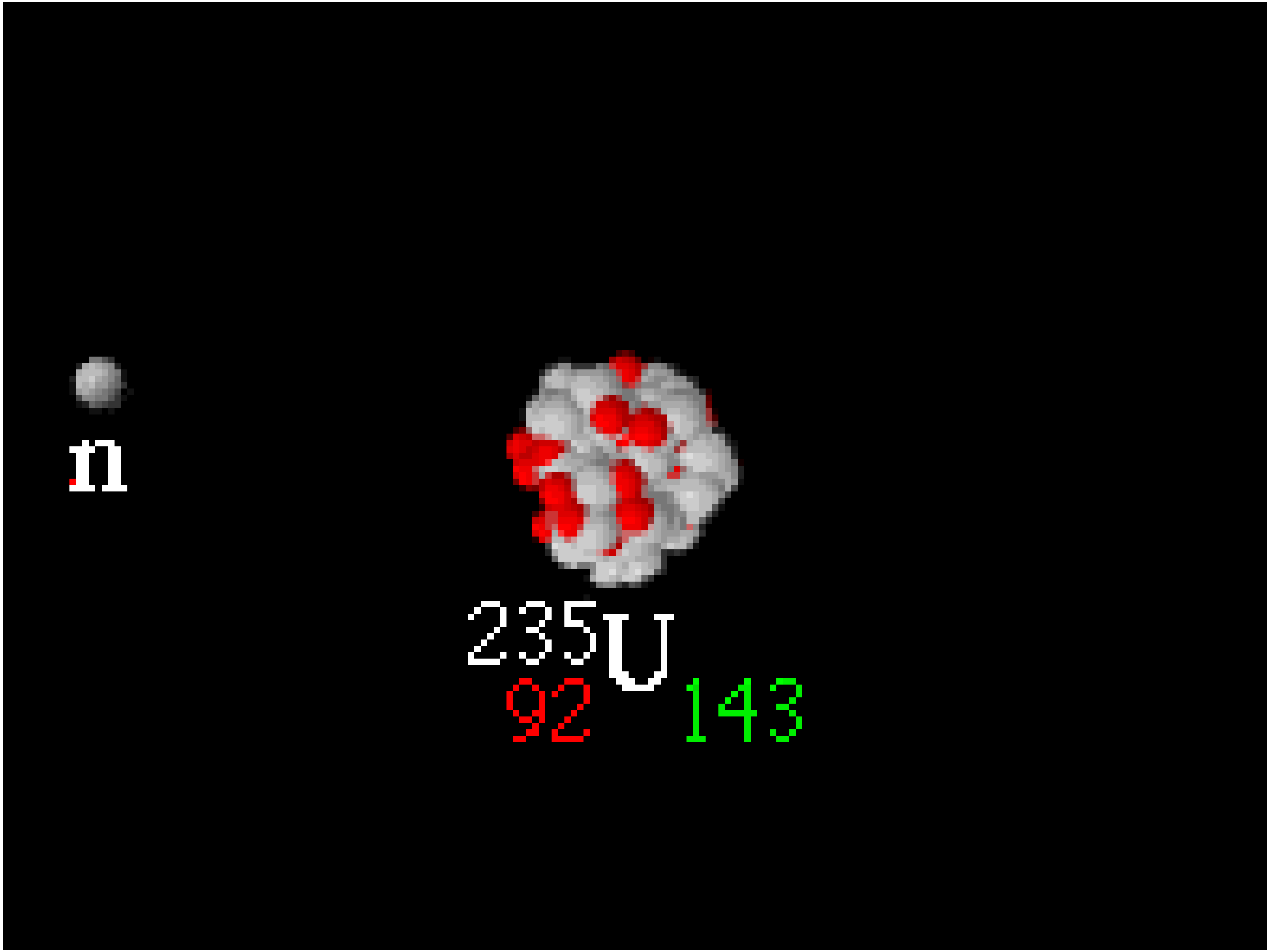

$^{4}_{2}He_{2}$



12C
4He



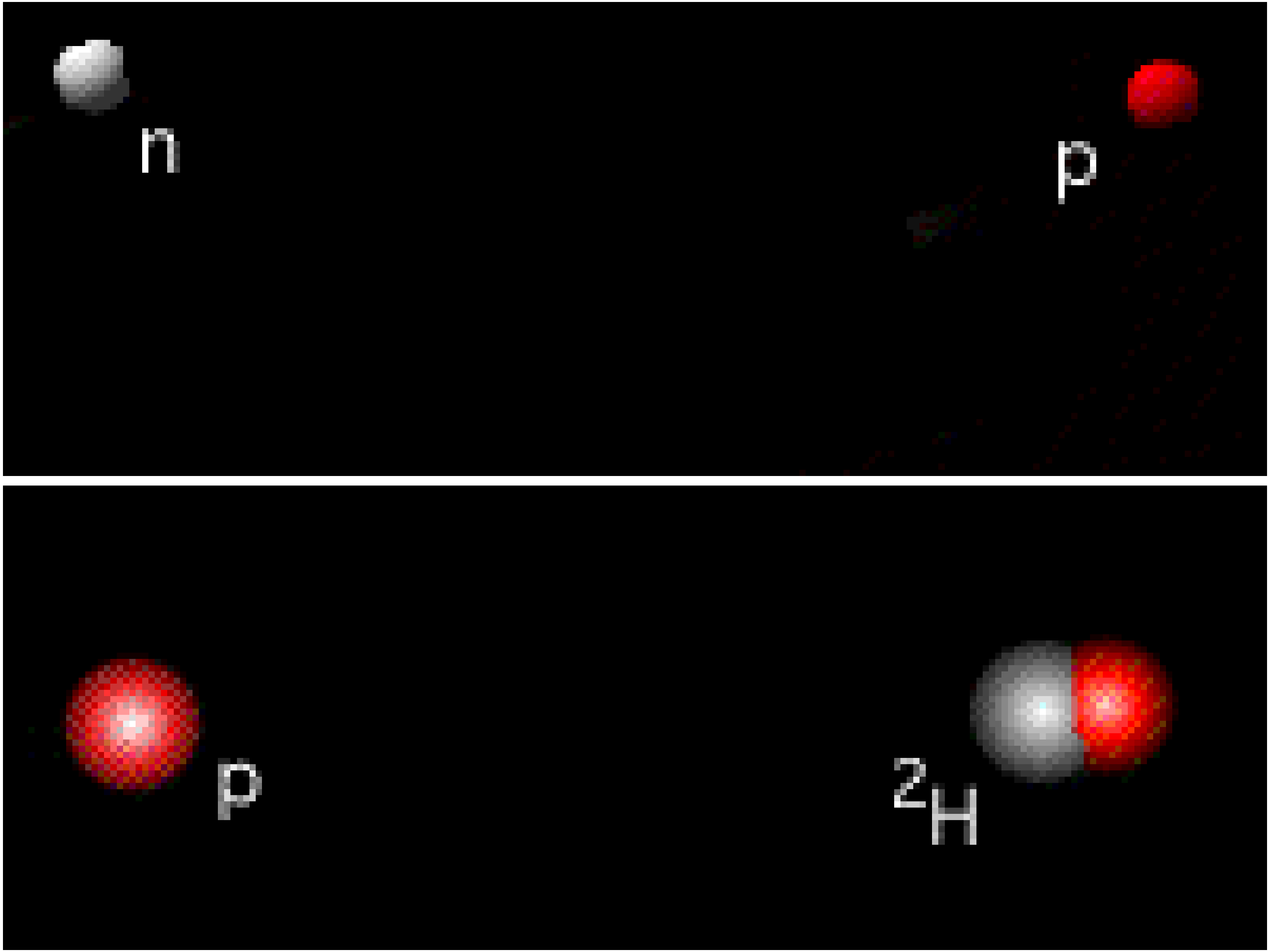
3He
4He
p
12C


n
235U
92
143

ESTRUTURA NUCLEAR

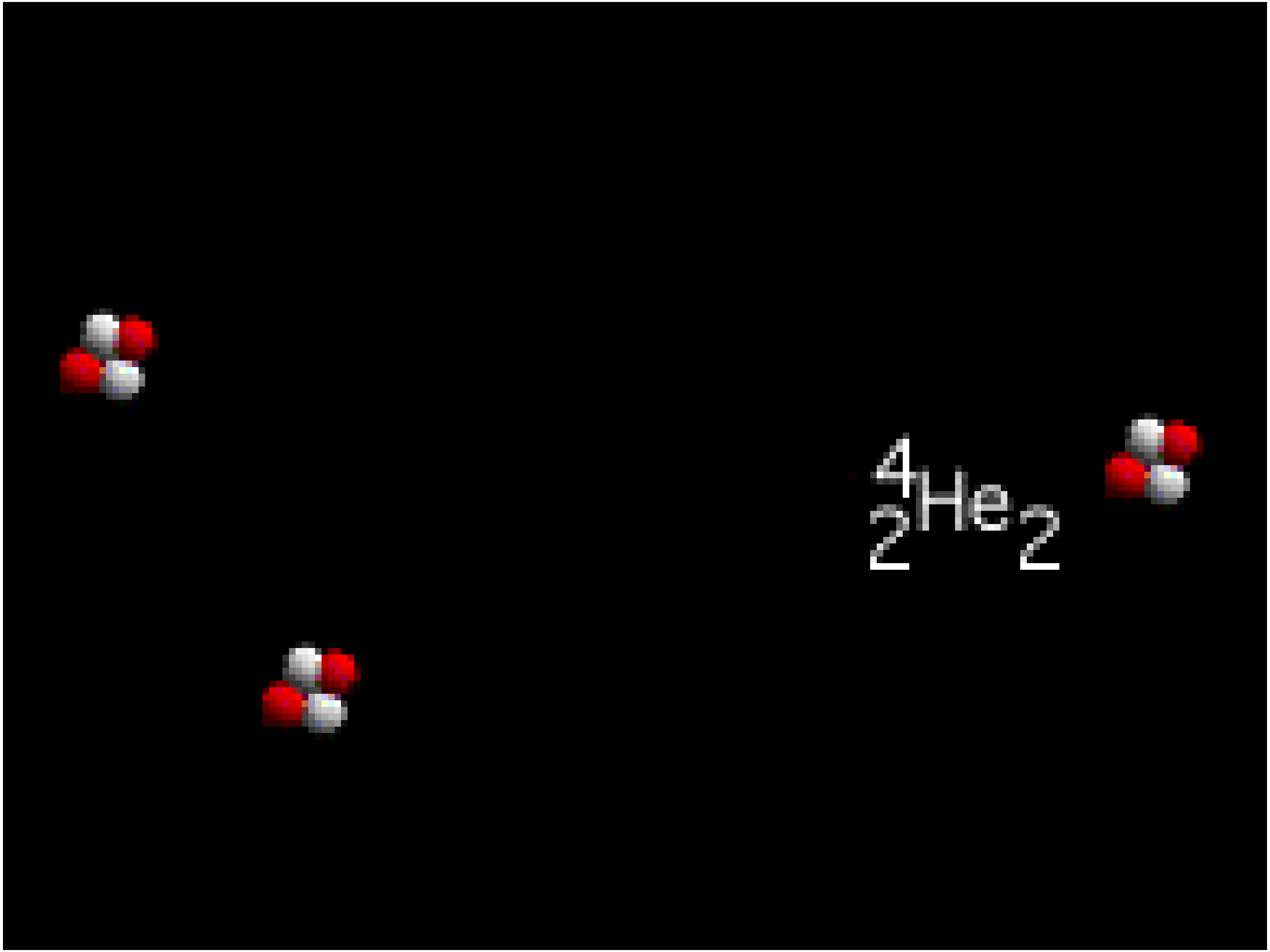
MECANISMO DE REAÇÕES

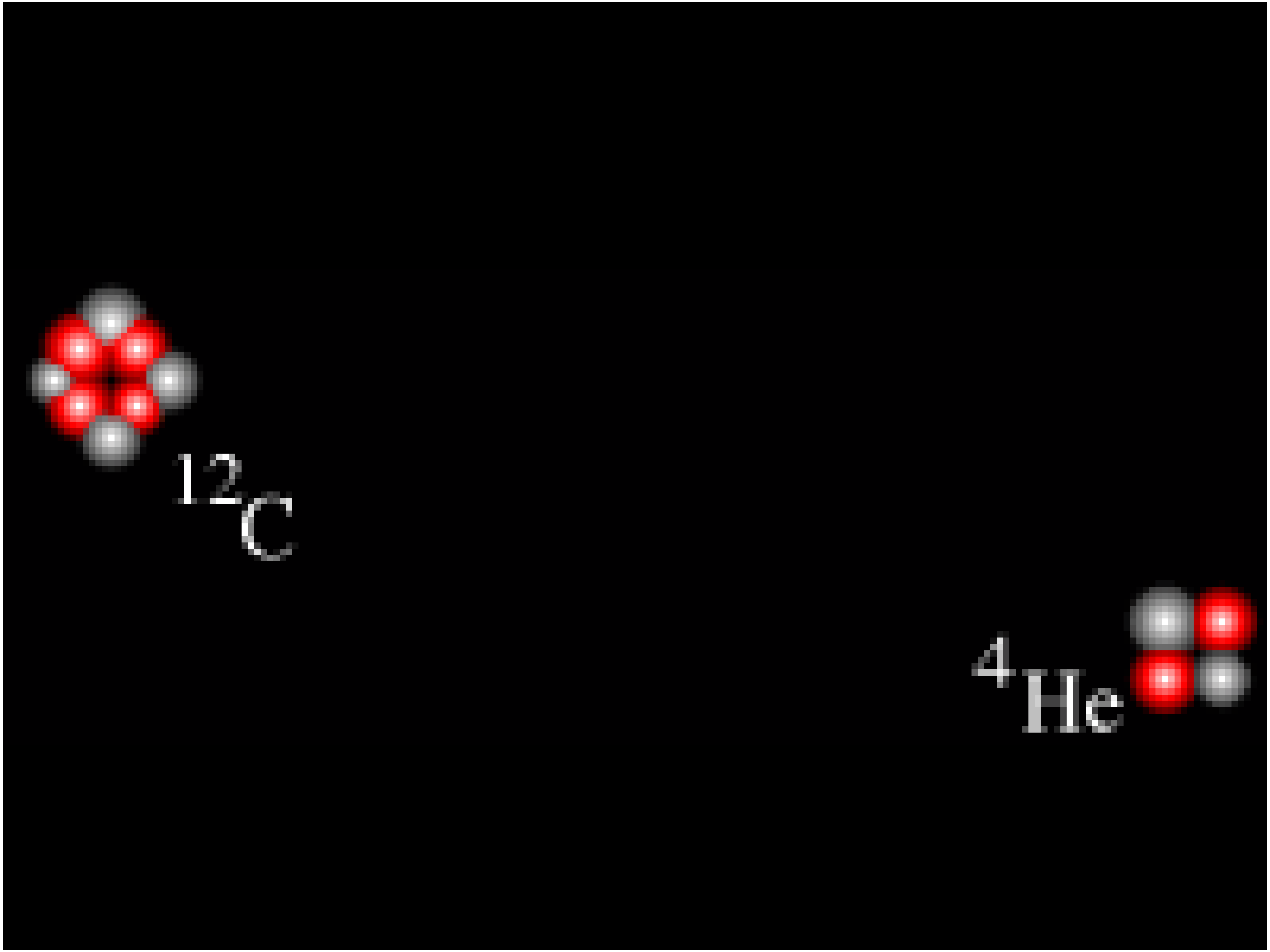
DINÂMICA DE REAÇÕES

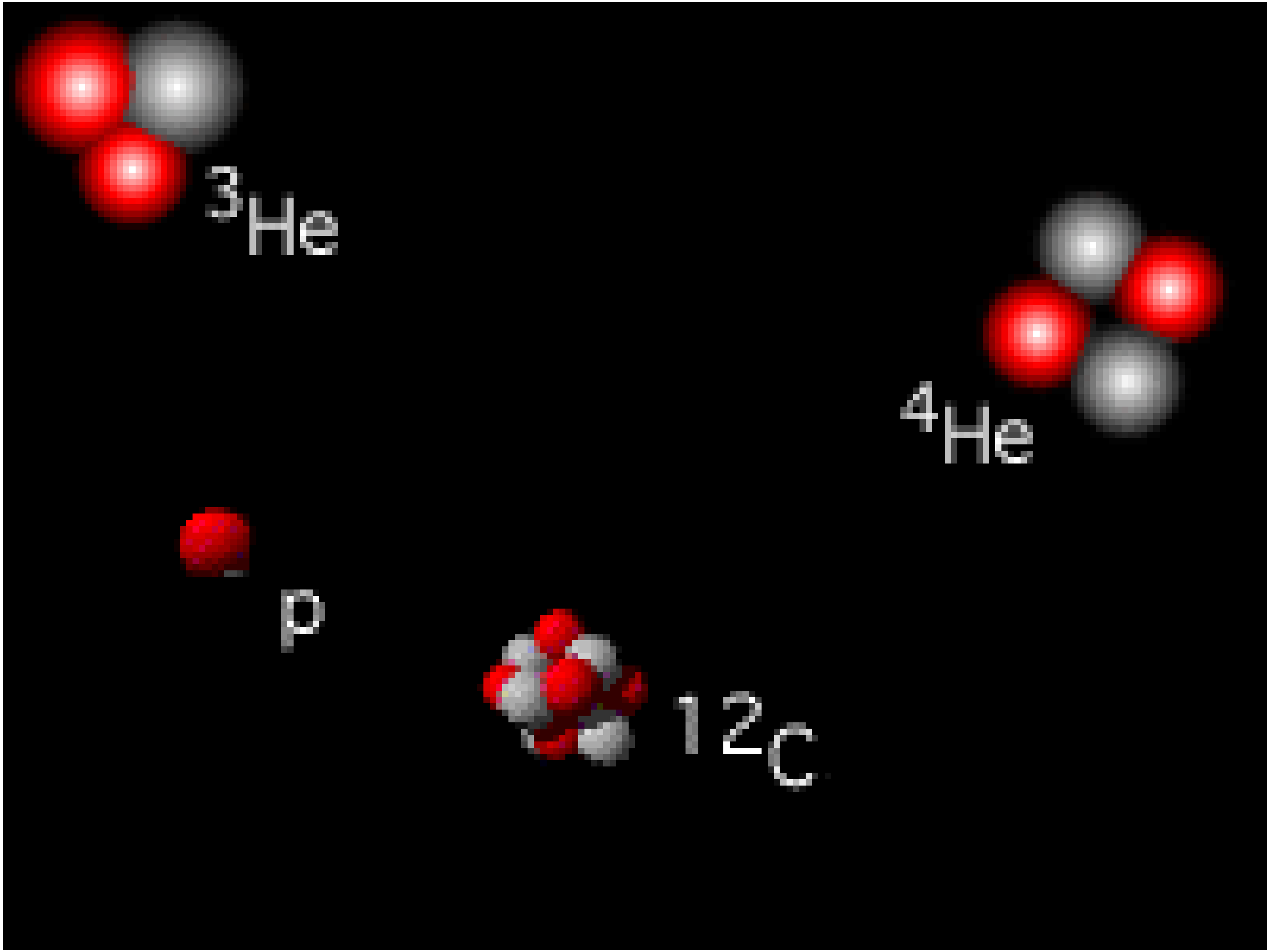

n
p
p
2H


$^{3}_{1}H_{2}$
$^{2}_{1}H_{1}$

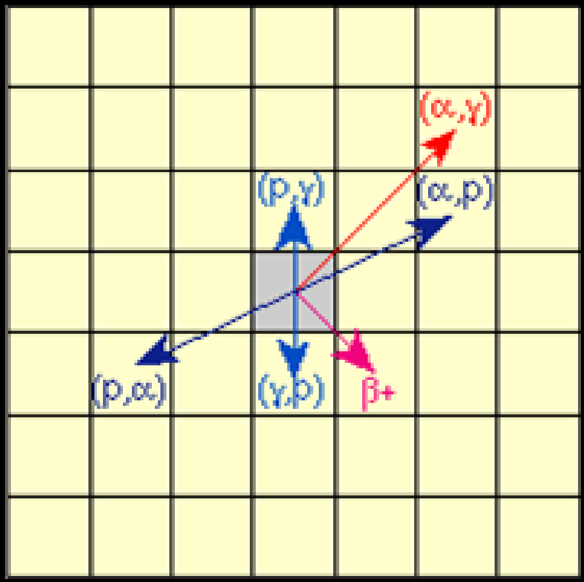

$^{3}_{1}H_{2}$
$^{2}_{1}H_{1}$


$^{4}_{2}He_2$

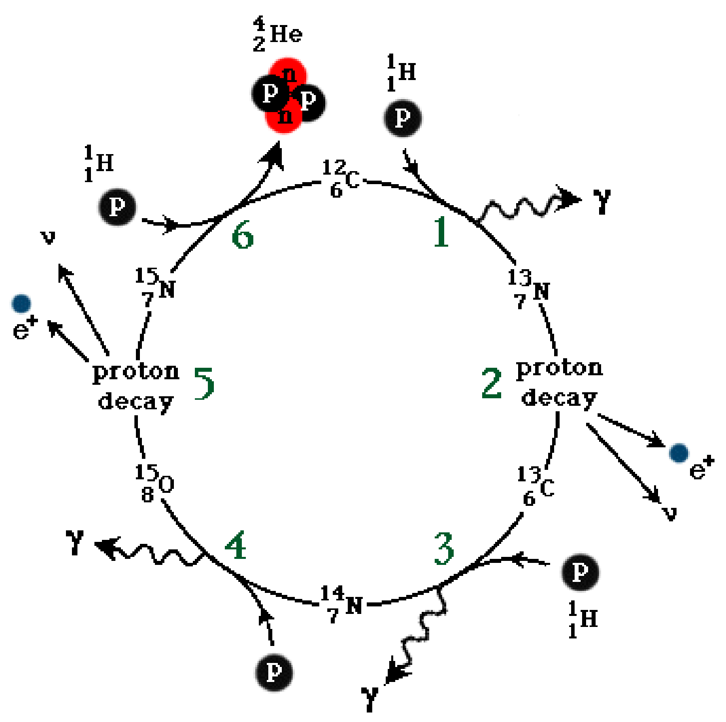

^{12}C
^{4}He

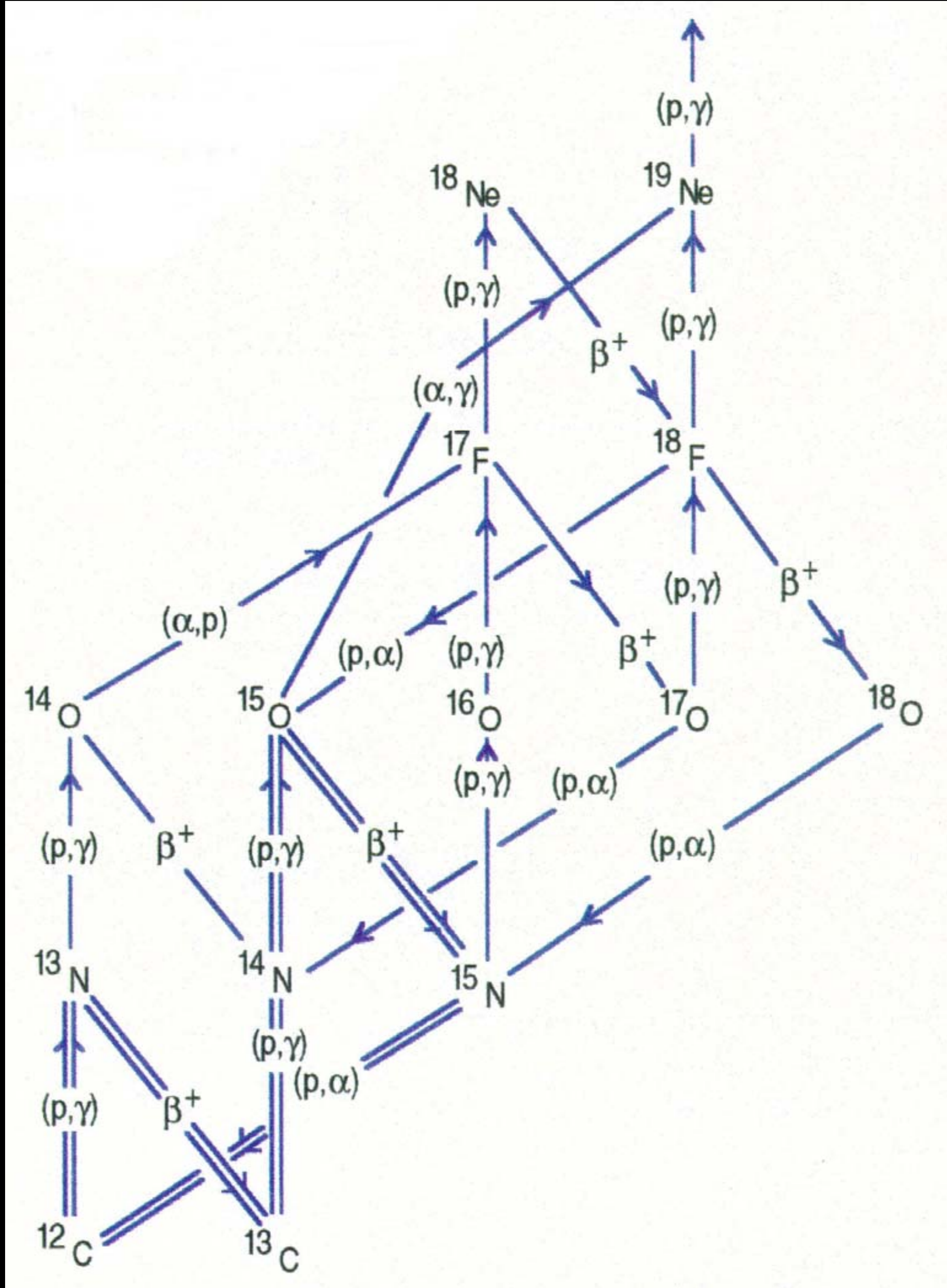


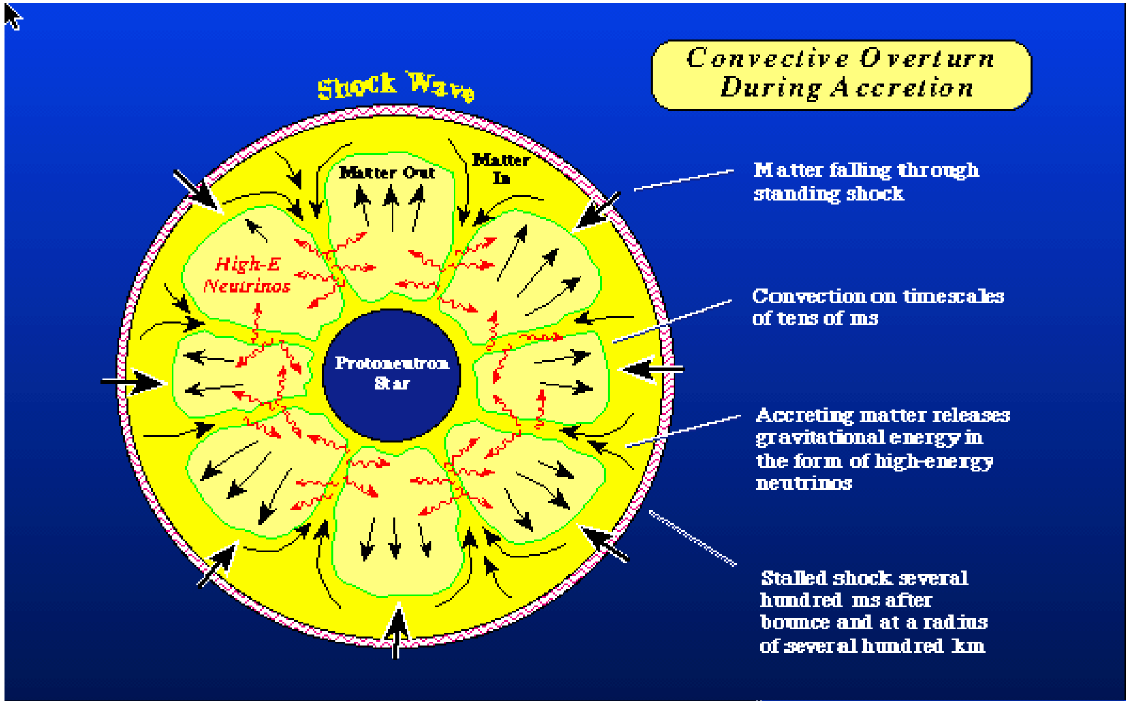
3He
4He
p
12C

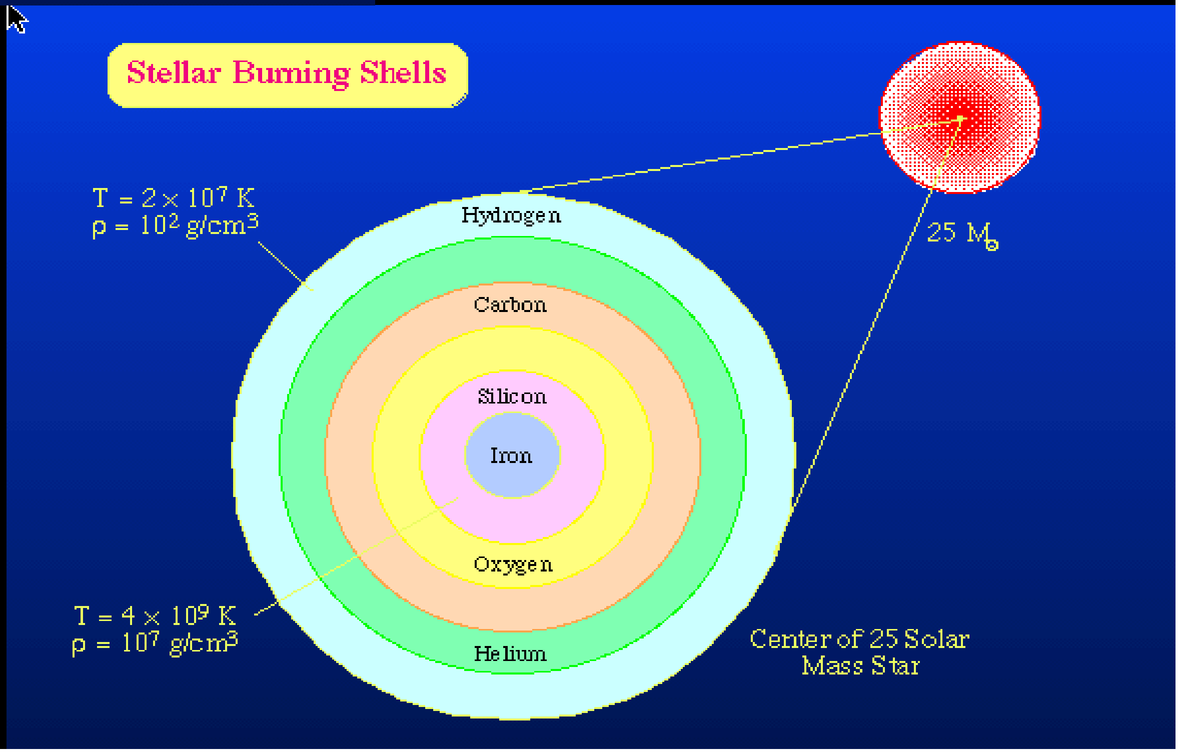


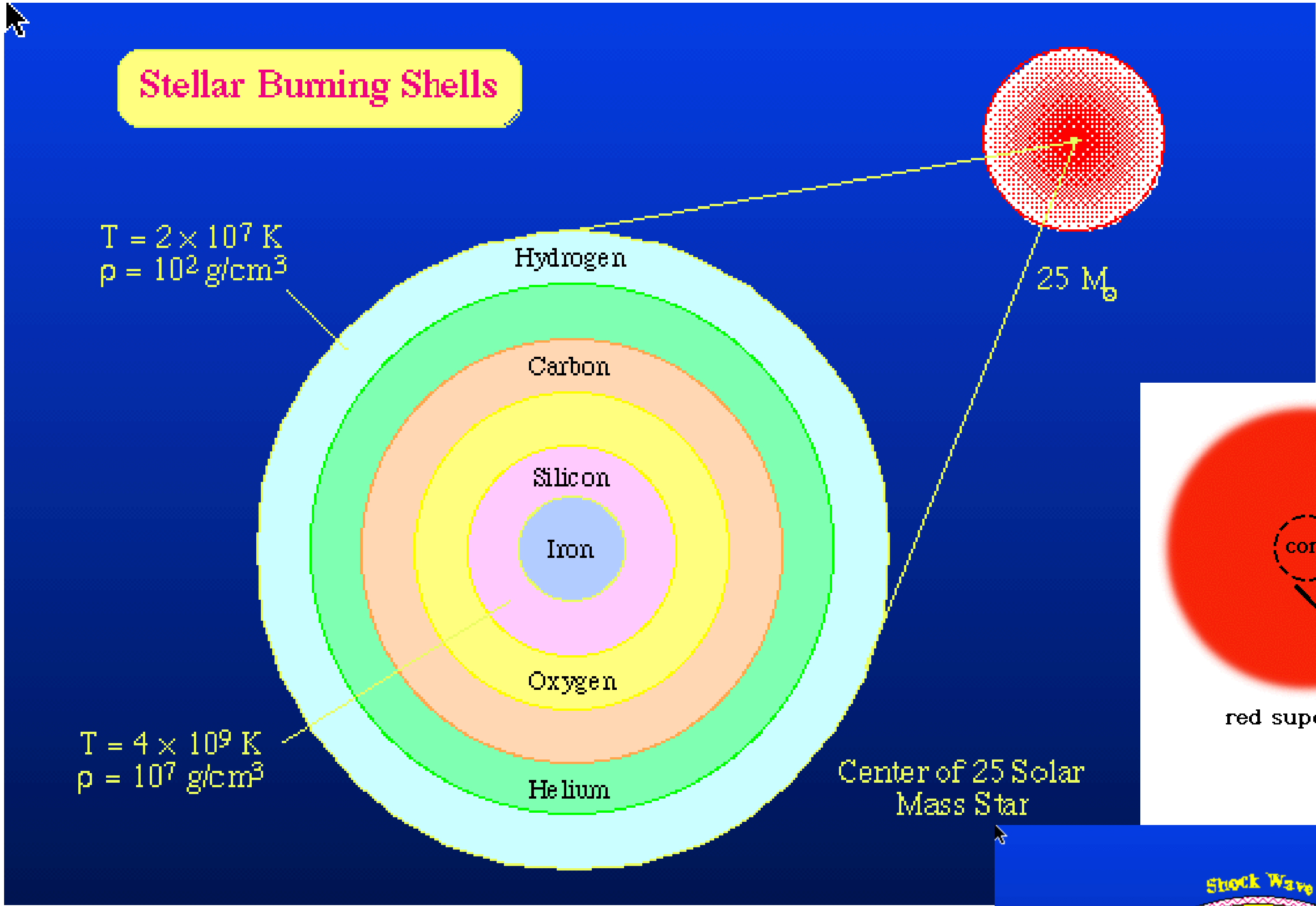
Carbon-Nitrogen-Oxygen Cycle



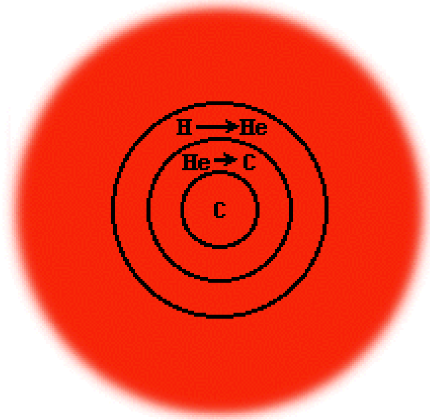



Convective Overturn
During Accretion
Shock Wave
Matter Out
Matter In
High-E Neutrinos
Protoneutron Star
Matter falling through standing shock
Convection on timescales of tens of ms
Accreting matter releases gravitational energy in the form of high-energy neutrinos
Stalled shock several hundred ms after bounce and at a radius of several hundred km

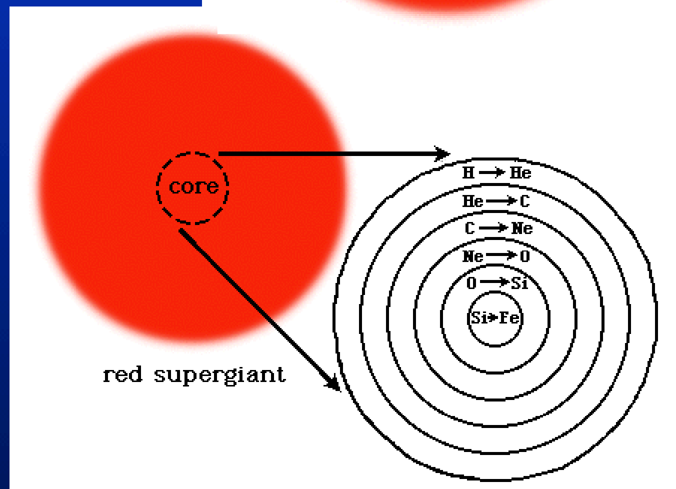

Stellar Burning Shells
$T = 2 \times 10^7$ K
$\rho = 10^2$ g/cm^3
Hydrogen
Helium
Carbon
Oxygen
Silicon
Iron
$T = 4 \times 10^9$ K
$\rho = 10^7$ g/cm^3
25 $M_\odot$
Center of 25 Solar Mass Star



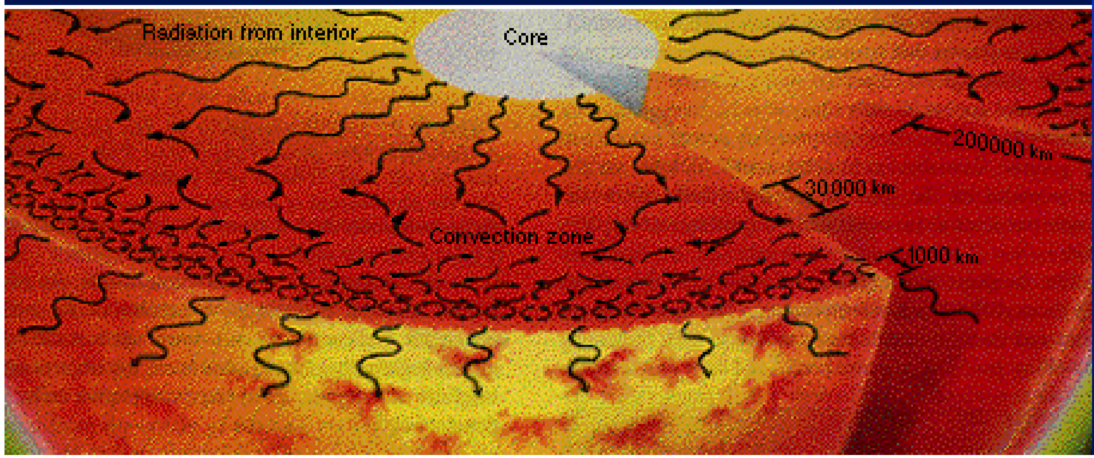
Stellar Burning Shells
$T = 2 \times 10^7$ K
$\rho = 10^2$ g/cm^3
Hydrogen
Carbon
Silicon
Iron
Oxygen
Helium
$T = 4 \times 10^9$ K
$\rho = 10^7$ g/cm^3
25 $M_\odot$
Center of 25 Solar Mass Star



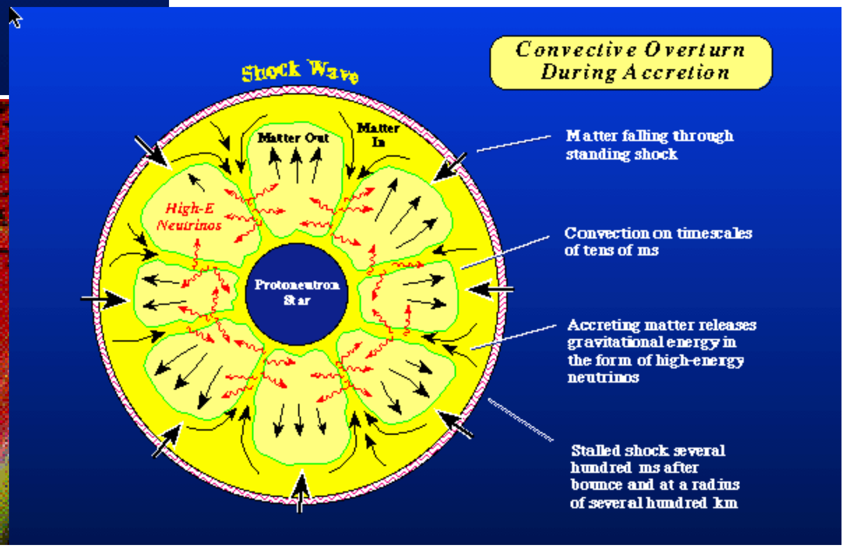
H → He
He → C
C



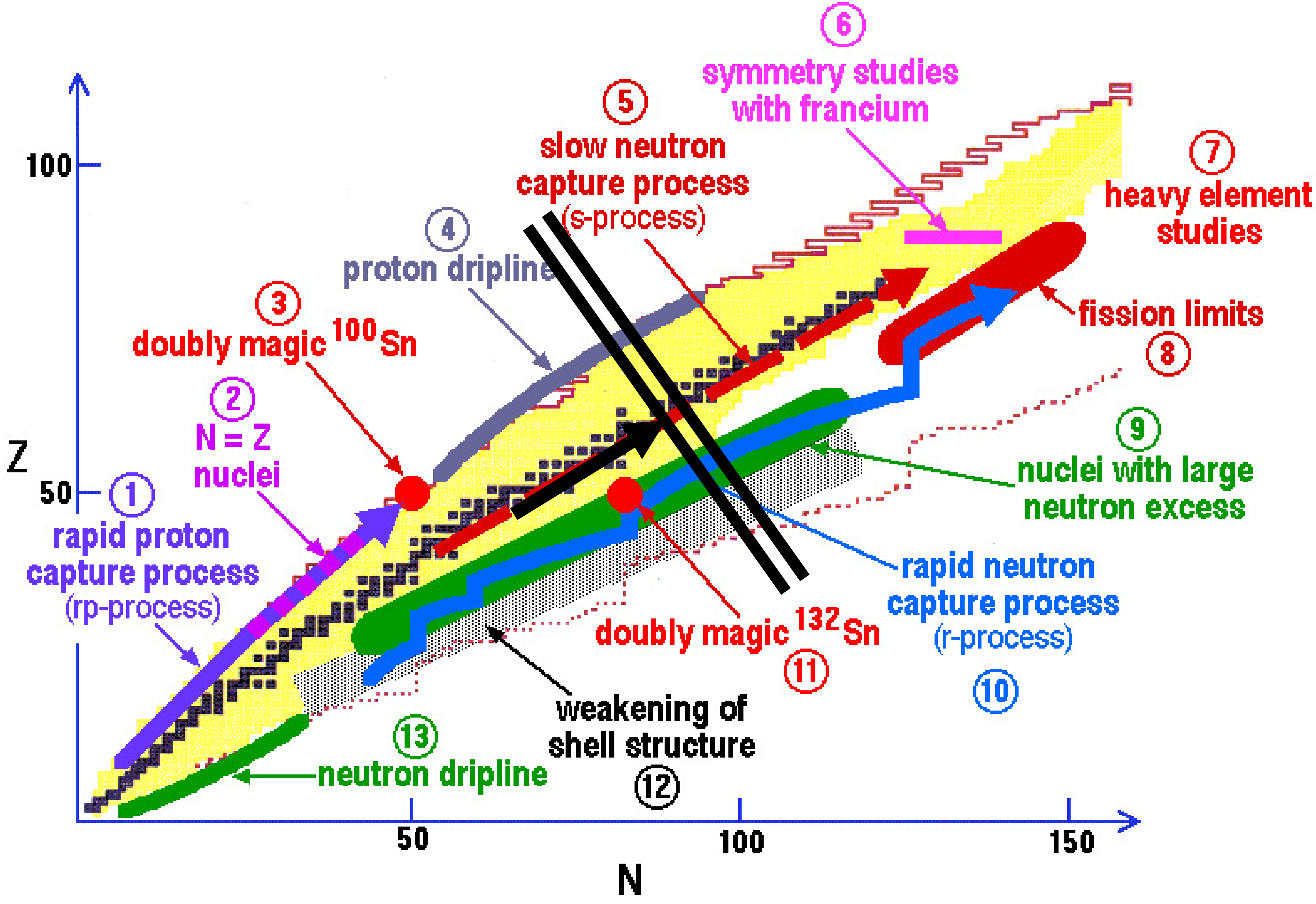
core
red supergiant
H → He
He → C
C → Ne
Ne → O
O → Si
Si → Fe



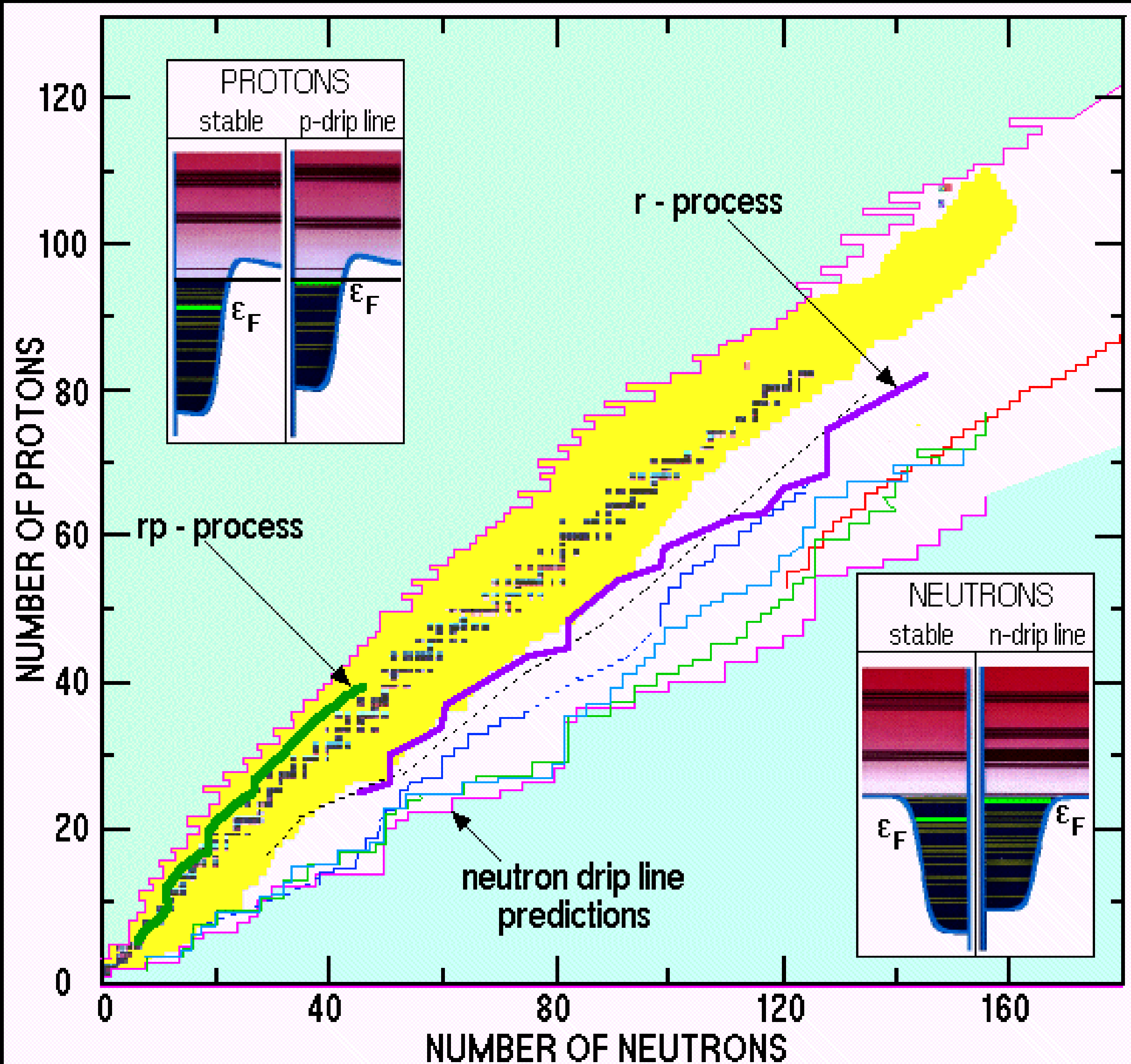
Radiation from interior
Core
200000 km
30,000 km
Convection zone
1000 km

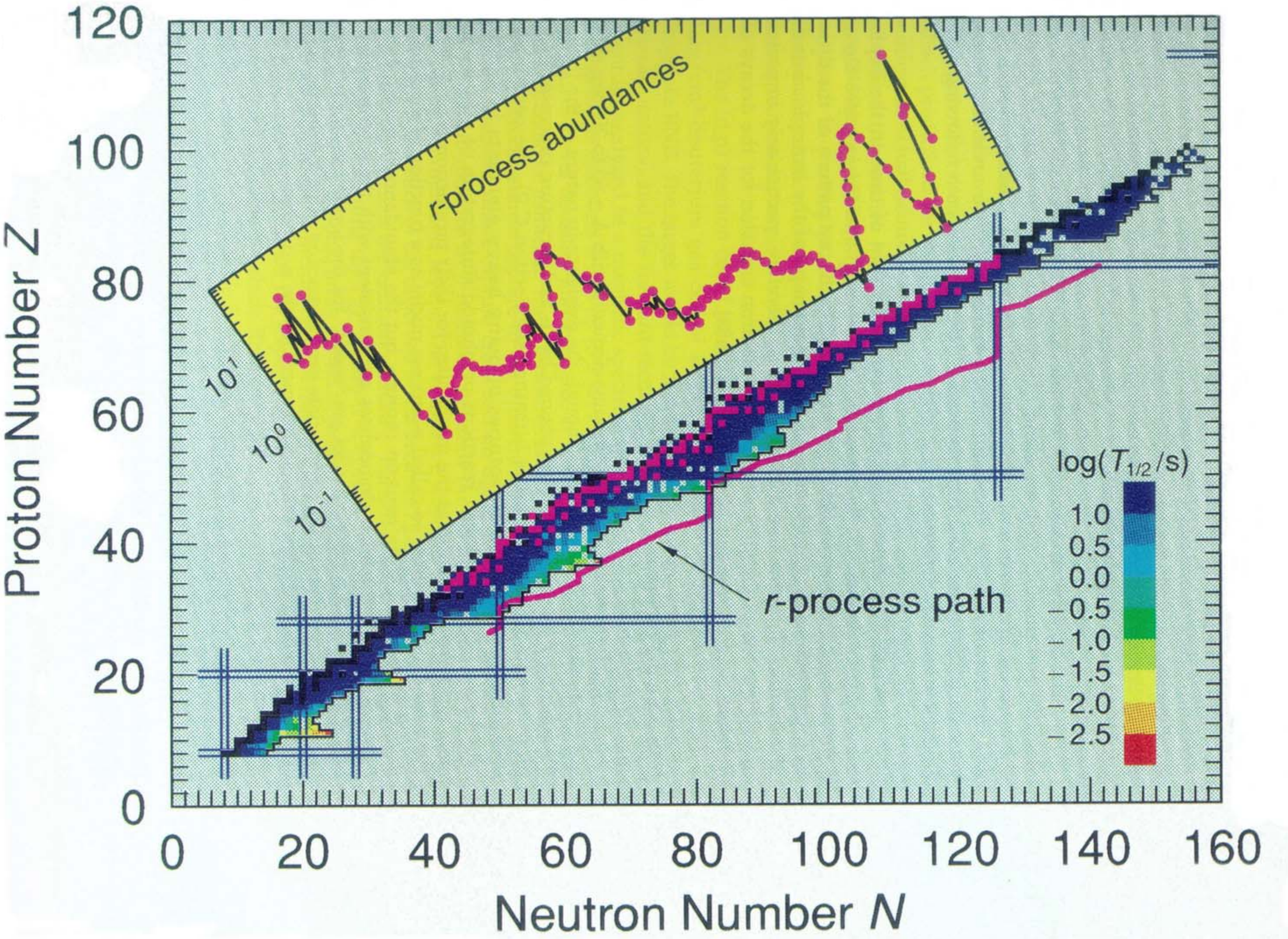


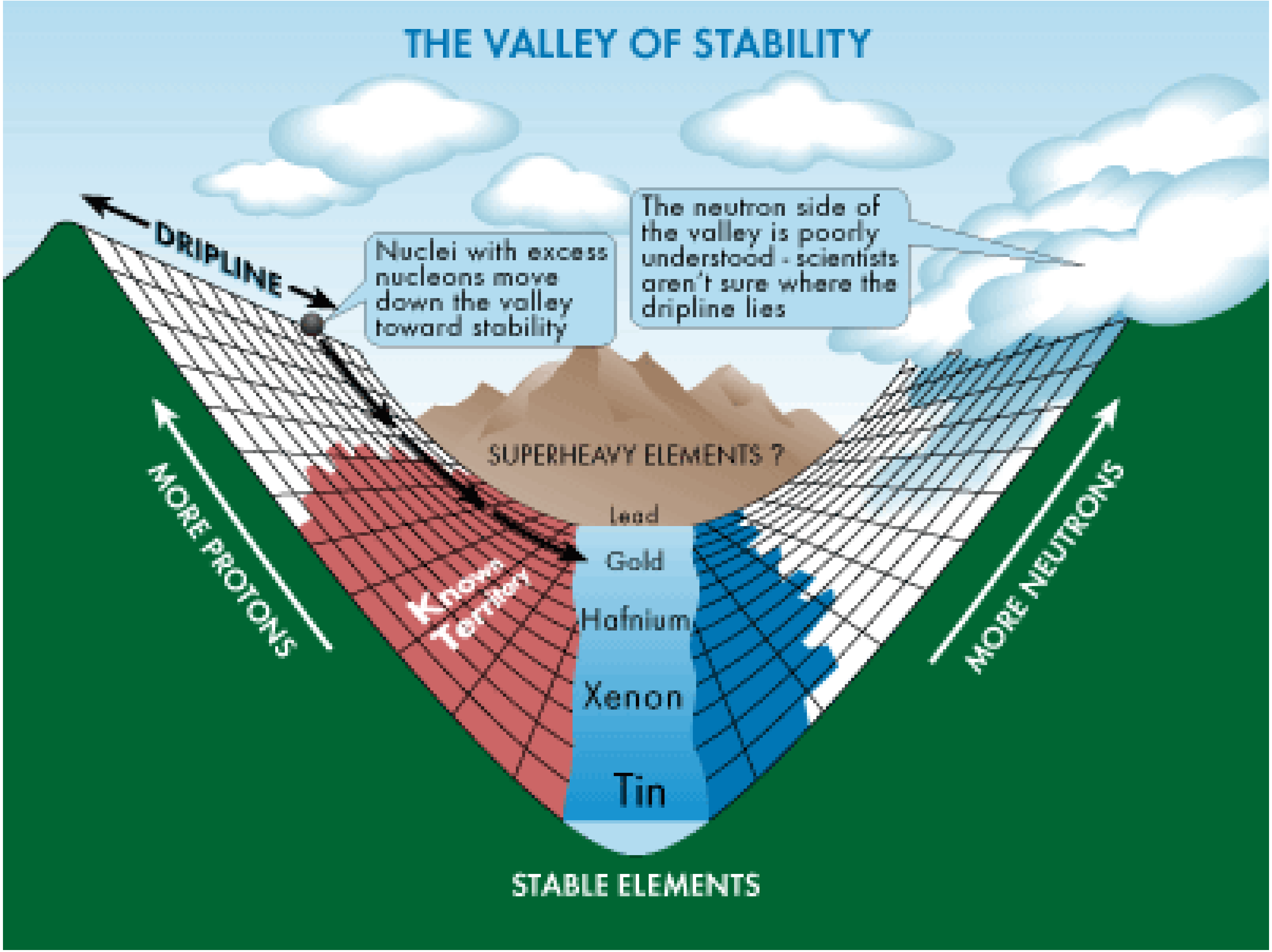
Convective Overturn During Accretion
Shock Wave
Matter Out
Matter In
High-E Neutrinos
Protoneutron Star
Matter falling through standing shock
Convection on timescales of tens of ms
Accreting matter releases gravitational energy in the form of high-energy neutrinos
Stalled shock several hundred ms after bounce and at a radius of several hundred km



100
50
Z
N
50
100
150
1
rapid proton
capture process
(rp-process)
2
N = Z
nuclei
3
doubly magic 100Sn
4
proton dripline
5
slow neutron
capture process
(s-process)
6
symmetry studies
with francium
7
heavy element
studies
fission limits
8
9
nuclei with large
neutron excess
rapid neutron
capture process
(r-process)
10
doubly magic 132Sn
11
weakening of
shell structure
12
13
neutron dripline


PROTONS
stable
p-drip line
εF
εF
r - process
rp - process
NEUTRONS
stable
n-drip line
εF
εF
neutron drip line
predictions
NUMBER OF PROTONS
NUMBER OF NEUTRONS
0
20
40
60
80
100
120
0
40
80
120
160


Proton Number Z
Neutron Number N
r-process abundances
r-process path
log(T1/2/s)
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
120
100
80
60
40
20
0
0
20
40
60
80
100
120
140
160


THE VALLEY OF STABILITY
DRIPLINE
Nuclei with excess nucleons move down the valley toward stability
The neutron side of the valley is poorly understood - scientists aren't sure where the dripline lies
SUPERHEAVY ELEMENTS ?
Lead
Gold
Hafnium
Xenon
Tin
Known Territory
MORE PROTONS
MORE NEUTRONS
STABLE ELEMENTS

BARREIRA COULOMBIANA

$$V_C = \frac{Z_A Z_a e^2}{R} = \frac{C}{R}$$

Para a colisão razante:

$$R_C = r_0 A_A^{1/3} + r_0 A_a^{1/3}$$
$$= r_0 (A_A^{1/3} + A_a^{1/3})$$

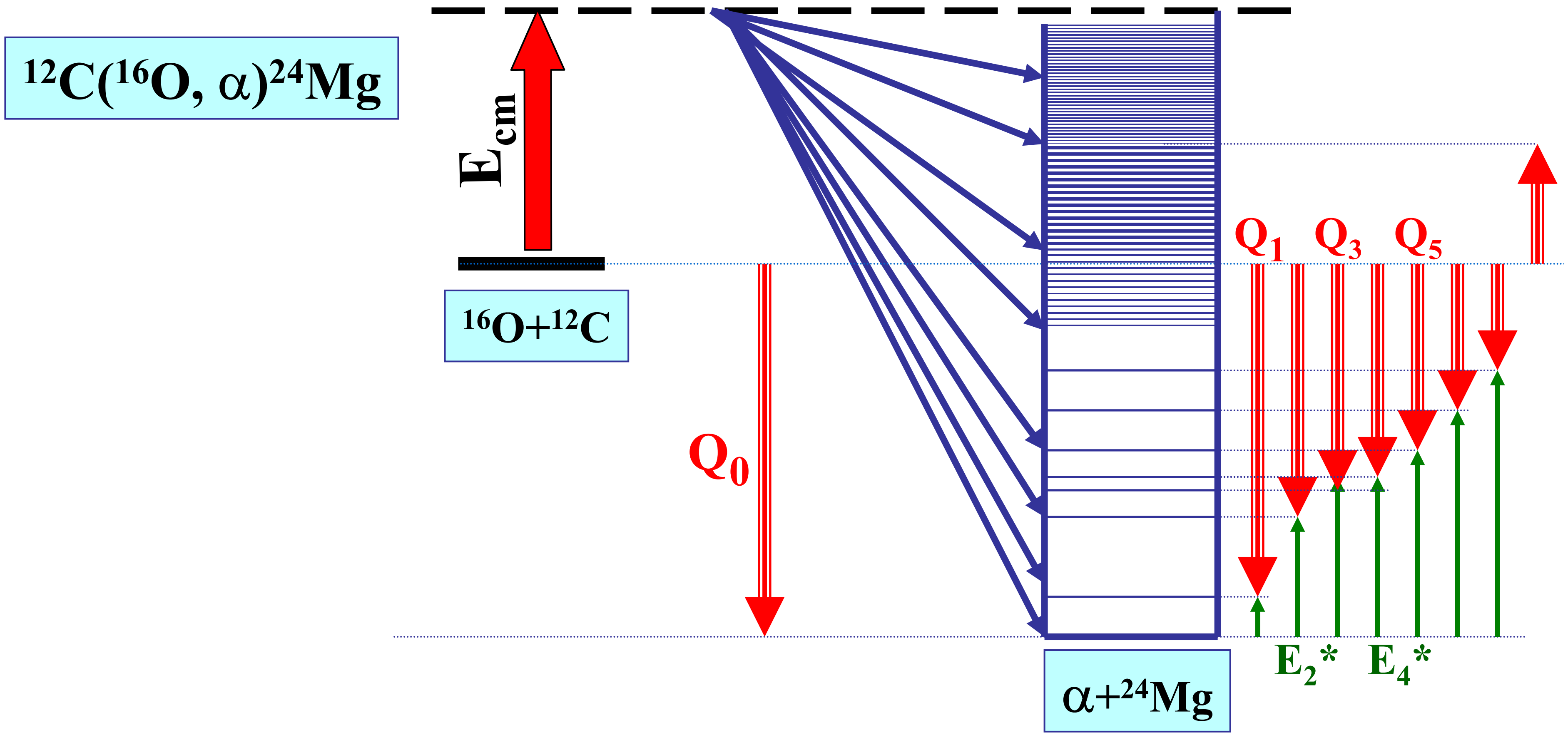
$$V_{Coul} = \frac{Z_A Z_a e^2}{r_0 (A_A^{1/3} + A_a^{1/3})}$$

$E_{cm} > V_{coul} \implies T_l(E_{cm}) = 1$

$E_{cm} < V_{coul} \implies T_l(E_{cm}) = 0$

e^2 = 1.44 MeV.fm
r_0 ~ 1.25 fm
V_{coul} (MeV)

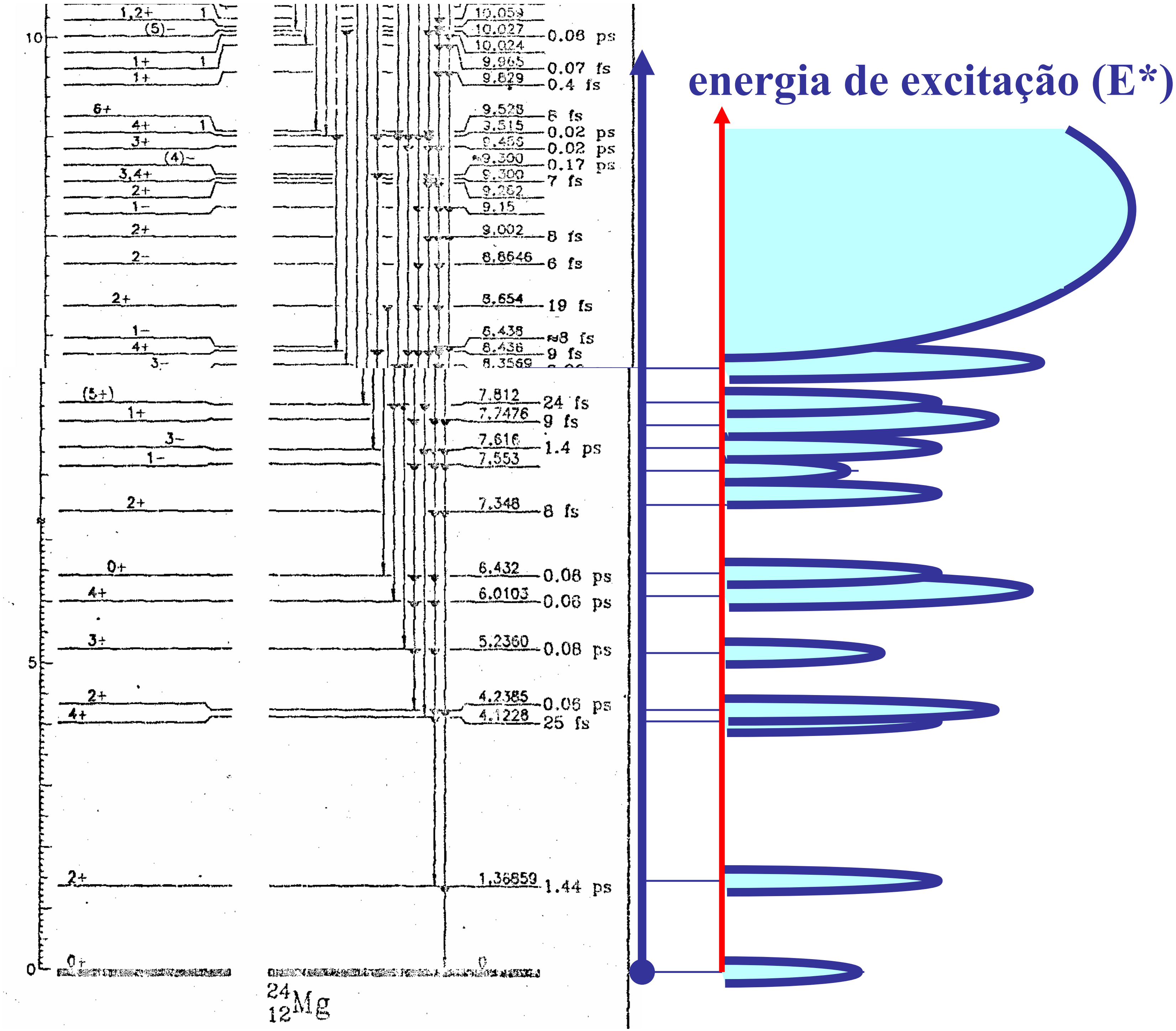
REAÇÃO DIRETA



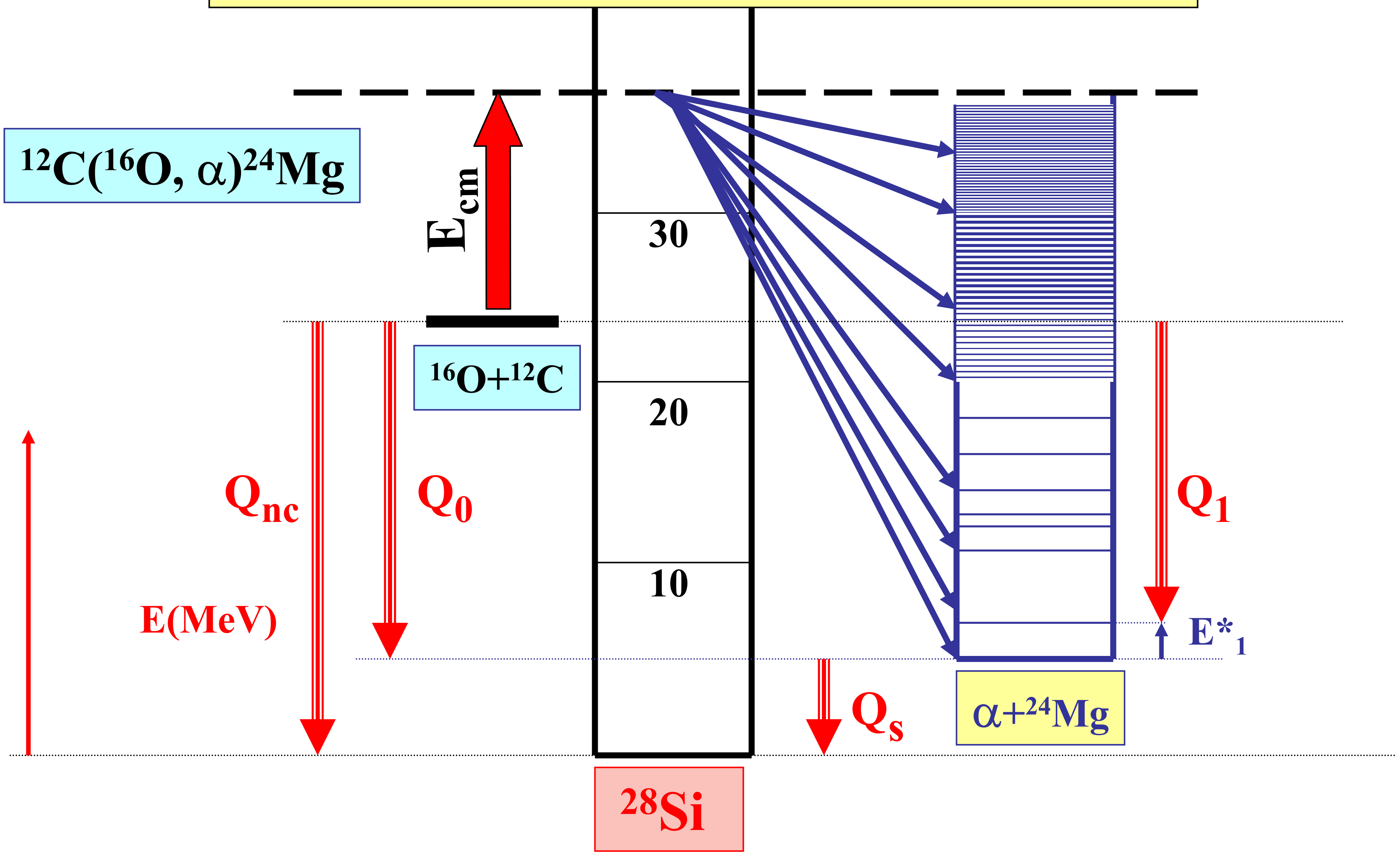
$E_{cm}(\alpha)$

$E^*_i[^{24}Mg(i)]$

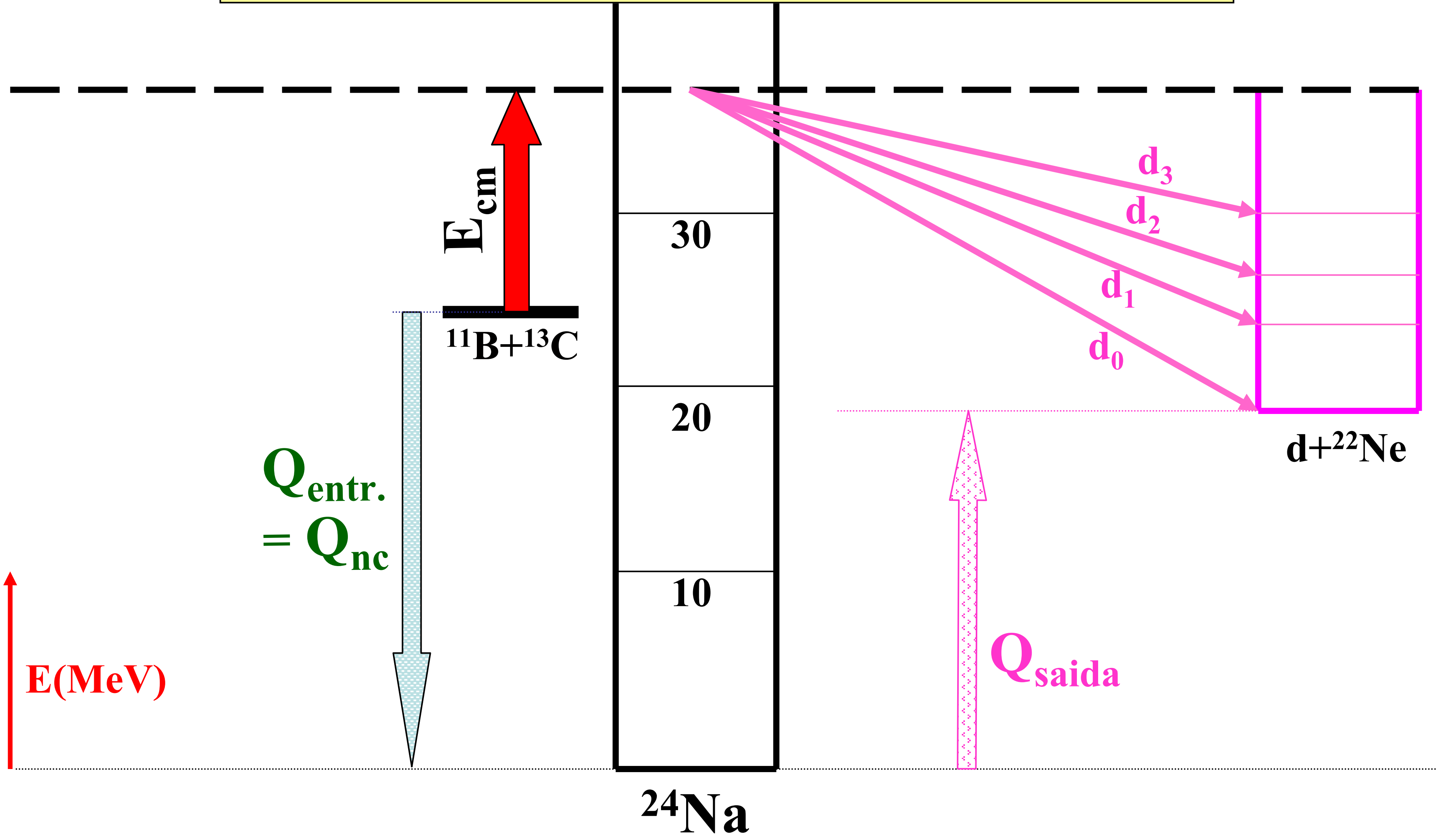
$Q_i[^{24}Mg(i)]$


energia de excitação (E*)
10
5
0
10.059
10.027
10.024
9.965
9.829
9.528
9.515
9.455
9.300
9.300
9.262
9.15
9.002
8.8646
8.654
8.438
8.436
8.3569
7.812
7.7476
7.616
7.553
7.348
6.432
6.0103
5.2360
4.2385
4.1228
1.36859
0
0.08 ps
0.07 fs
0.4 fs
8 fs
0.02 ps
0.02 ps
0.17 ps
7 fs
8 fs
6 fs
19 fs
≈8 fs
9 fs
24 fs
9 fs
1.4 ps
8 fs
0.08 ps
0.08 ps
0.08 ps
0.06 ps
25 fs
1.44 ps
$^{24}_{12}Mg$

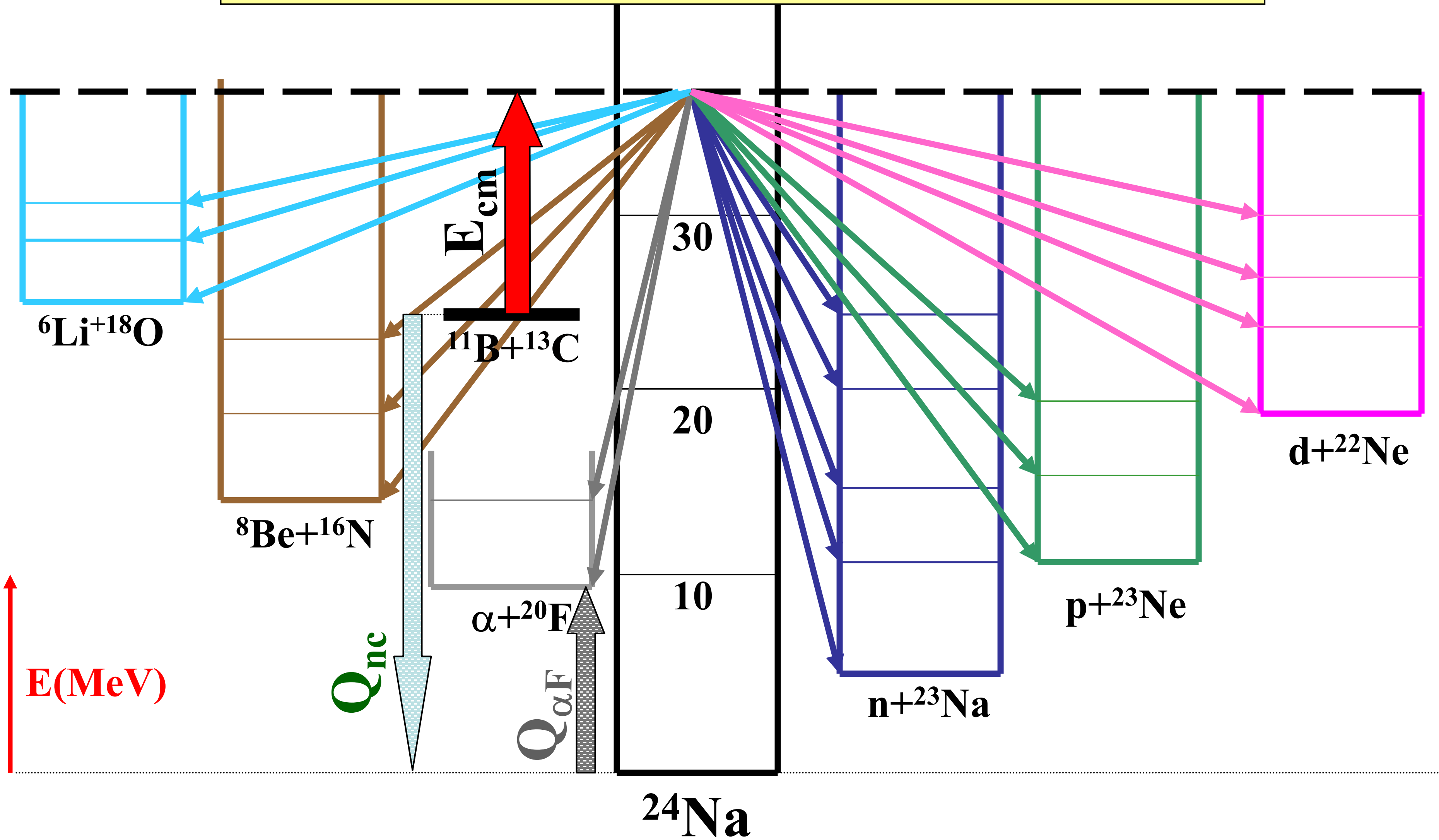
REAÇÃO VIA NÚCLEO COMPOSTO

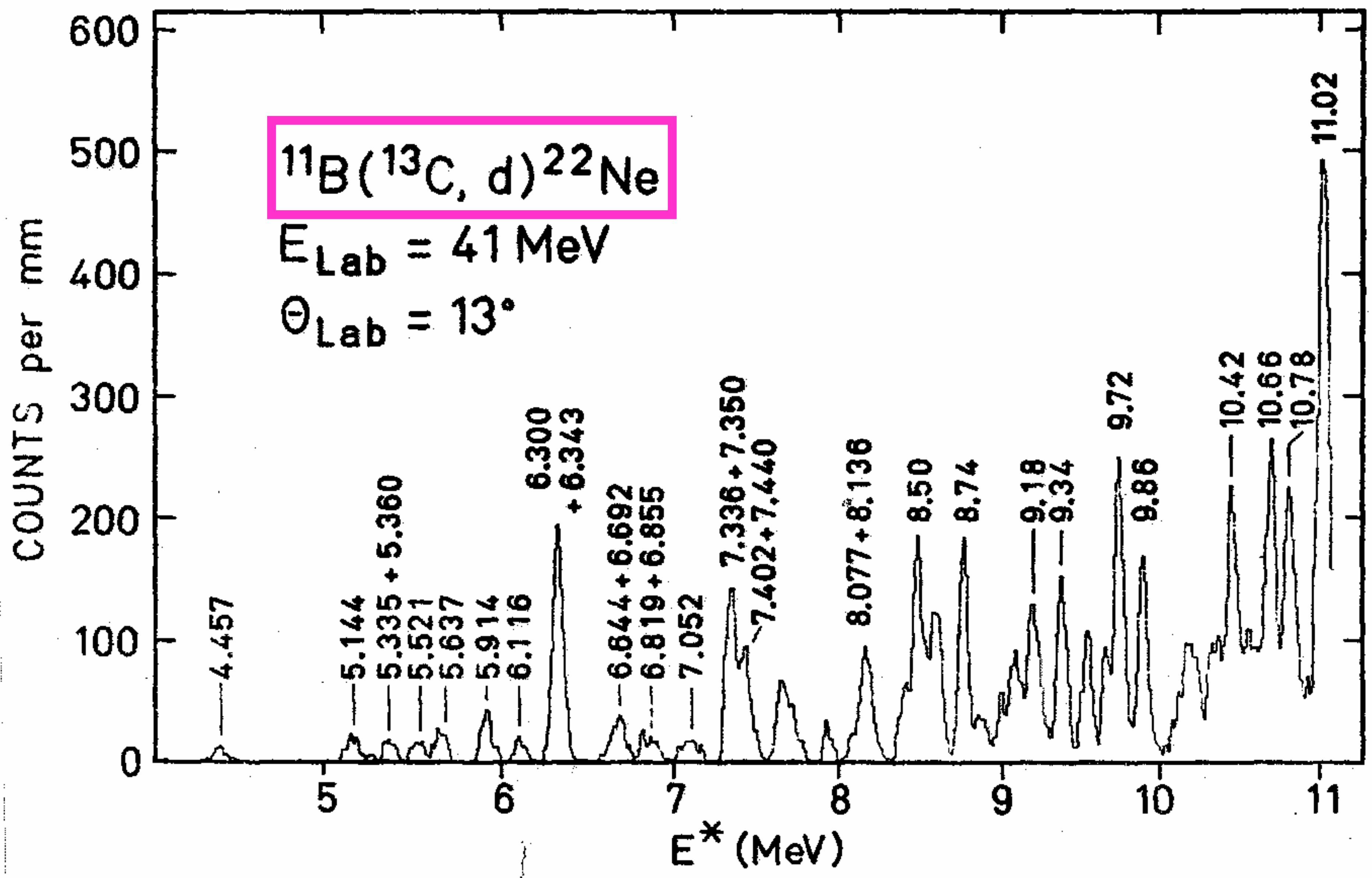


REAÇÃO VIA NÚCLEO COMPOSTO



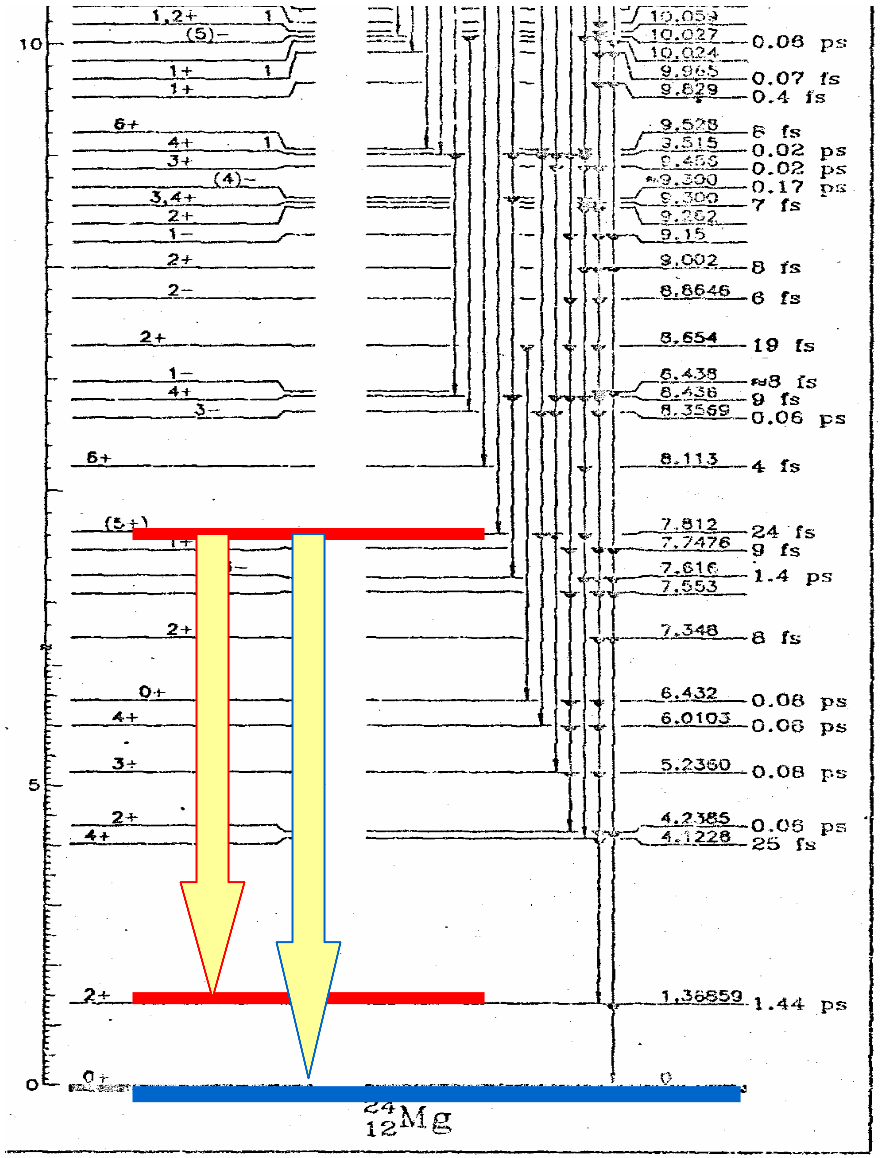
REAÇÕES VIA NÚCLEO COMPOSTO



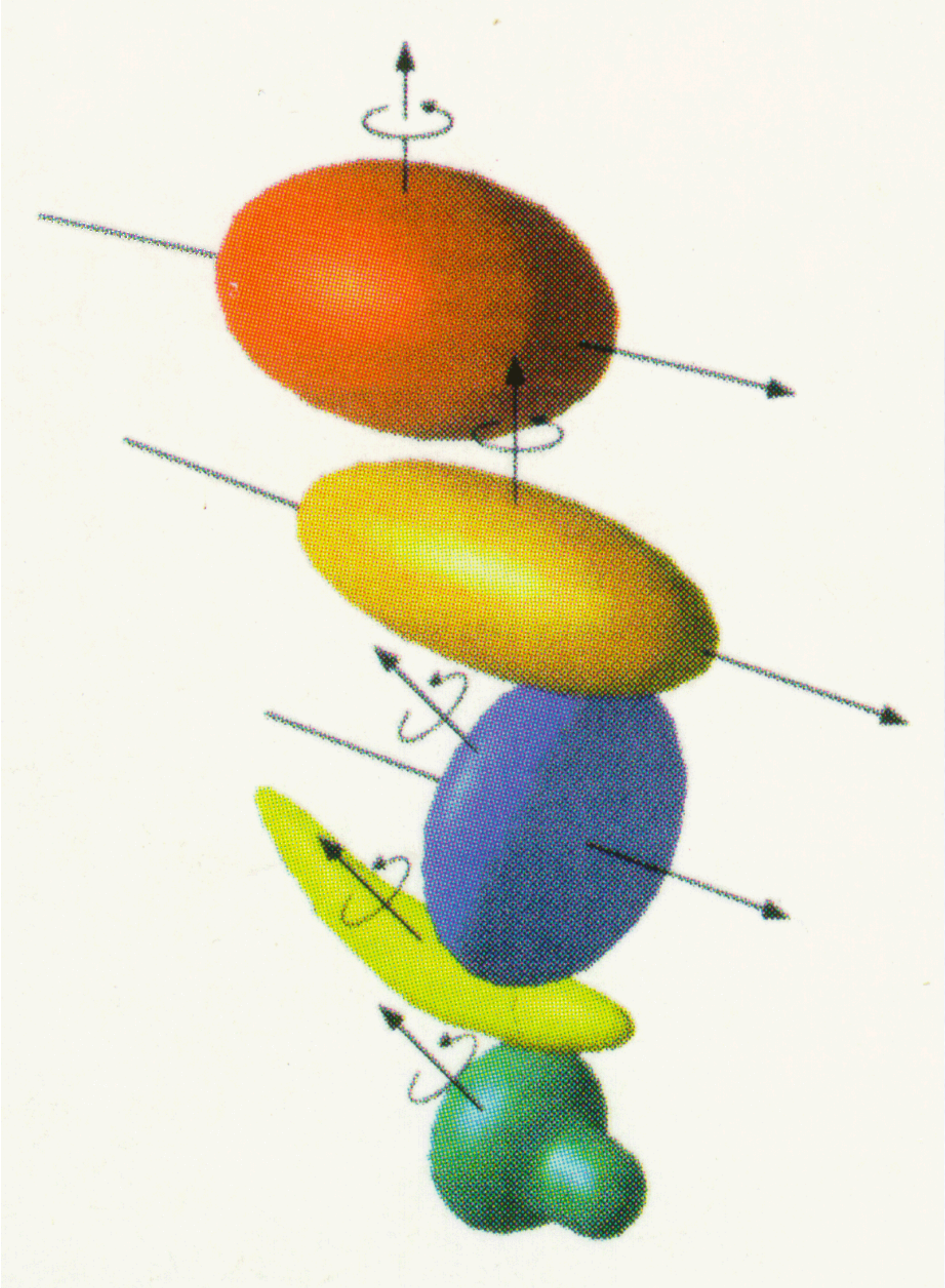


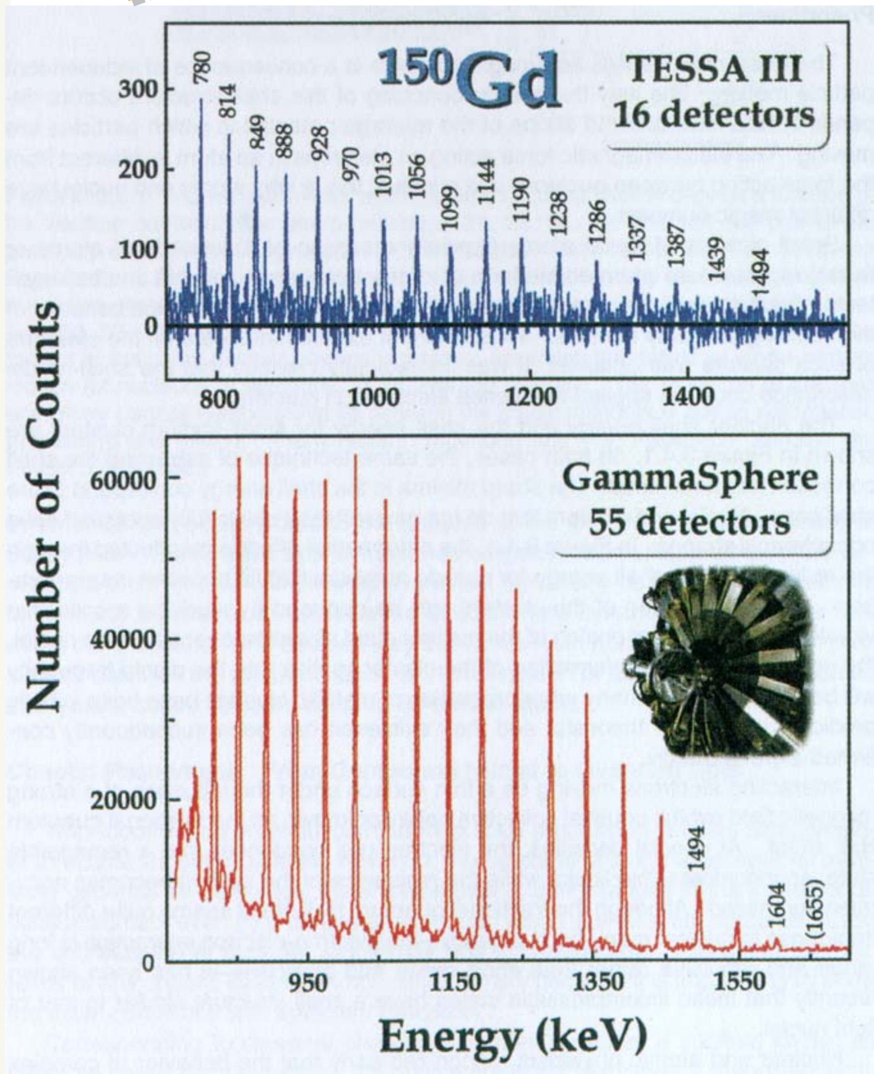
$^{11}B(^{13}C, d)^{22}Ne$
E_{Lab} = 41 MeV
Θ_{Lab} = 13°
COUNTS per mm
E* (MeV)
0
100
200
300
400
500
600
5
6
7
8
9
10
11
4.457
5.144
5.335 + 5.360
5.521
5.637
5.914
6.116
6.300
+ 6.343
6.644 + 6.692
6.819 + 6.855
7.052
7.336 + 7.350
7.402 + 7.440
8.077 + 8.136
8.50
8.74
9.18
9.34
9.72
9.86
10.42
10.66
10.78
11.02

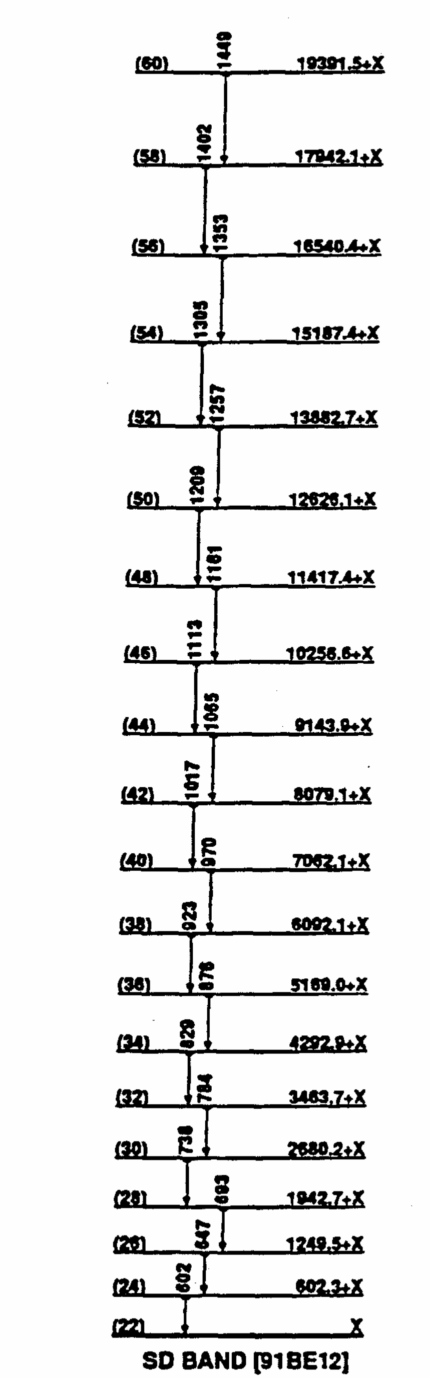
DECAIMENTO GAMMA (γ)



INCLUIR DIAGRAMA DE NIVEIS





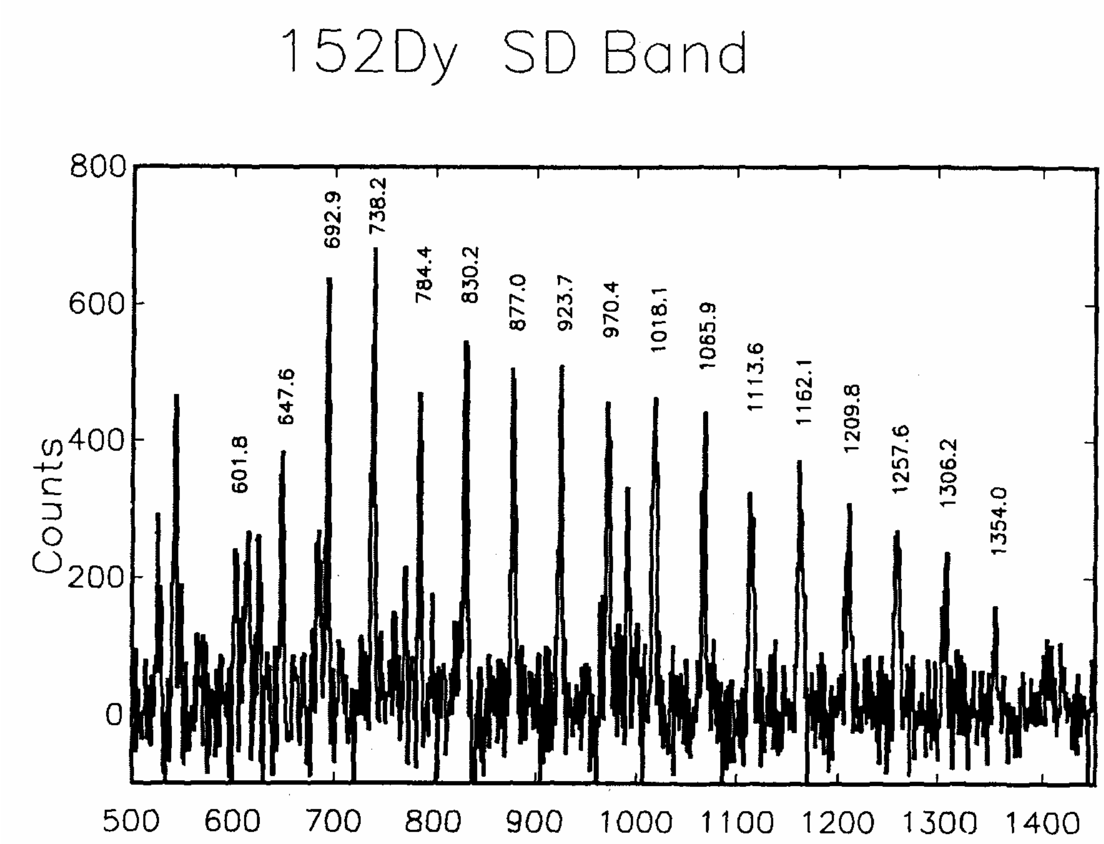


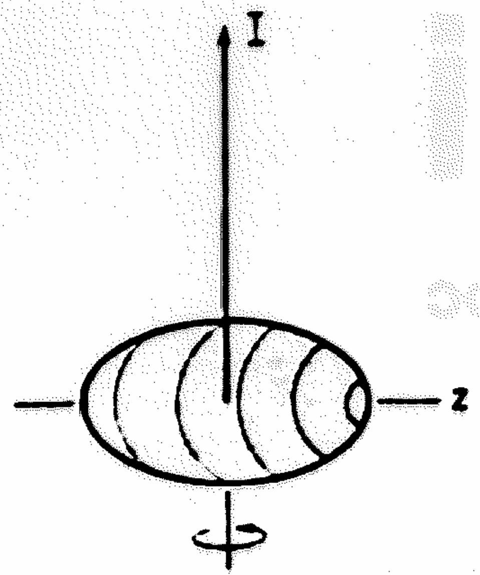
$^{152}_{66}$Dy Rotational Band

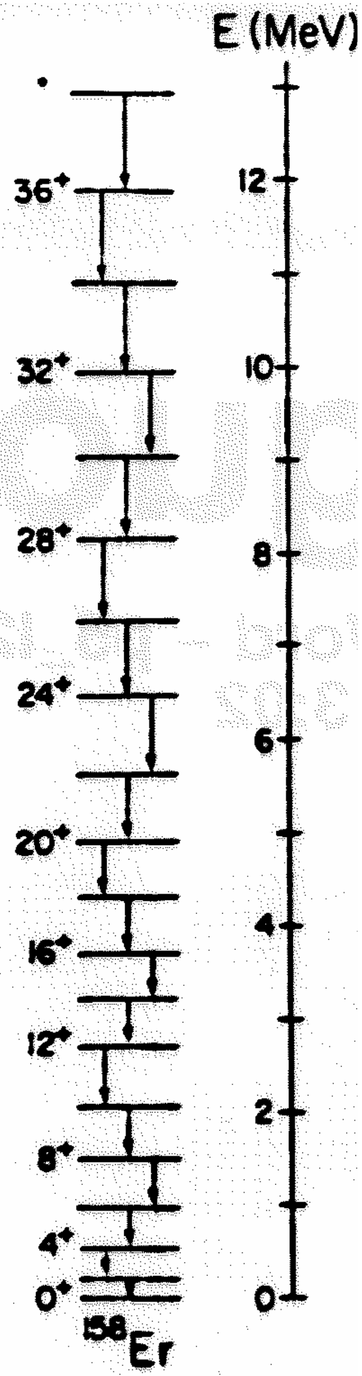
$$E^{\gamma}_{rot} = \frac{\hbar^2}{2\Im}[L(L+1)]$$

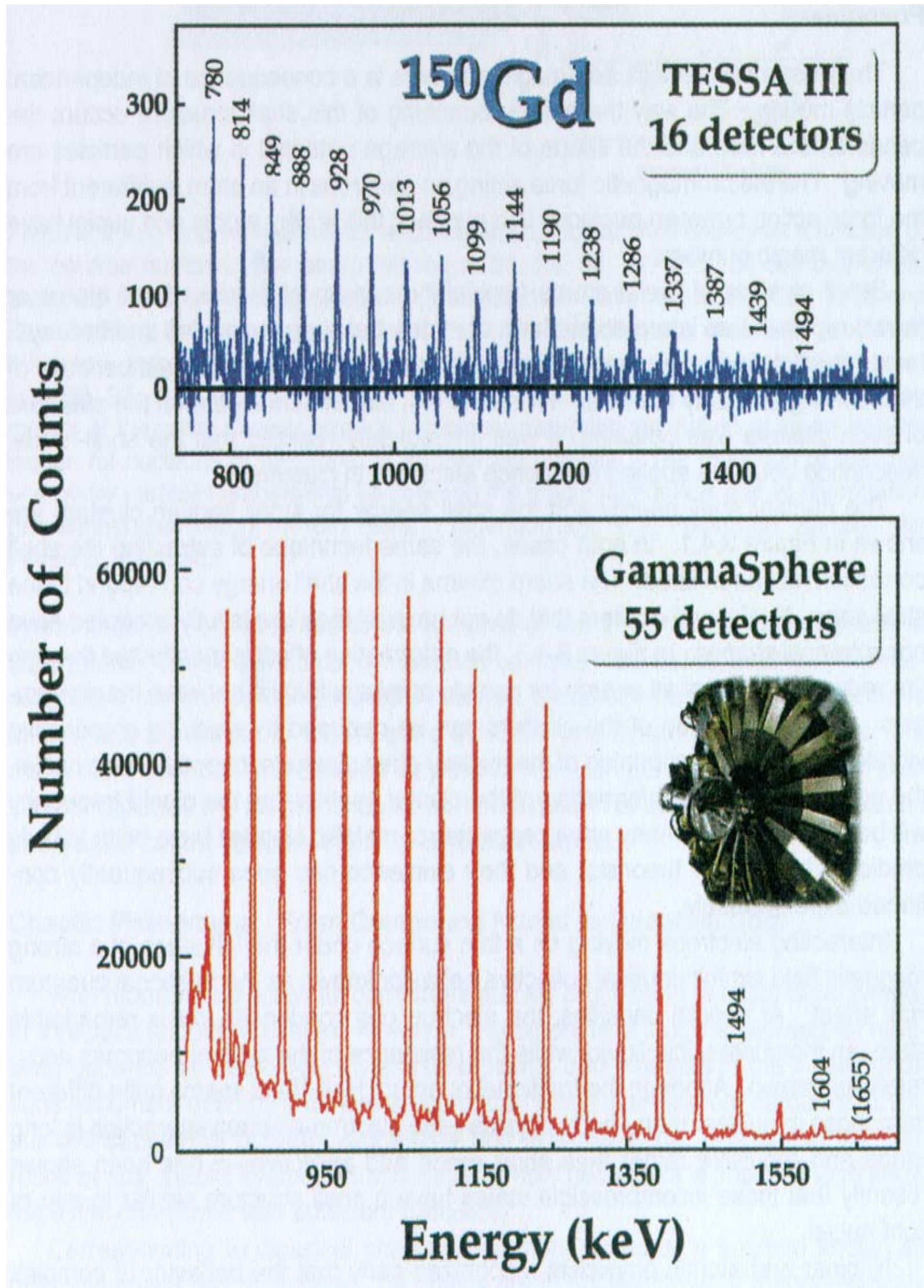
$$E^{\gamma}_{(L+2)\to L} = \frac{\hbar^2}{\Im}(2L+3) = E^{\gamma}_{(L+2)}$$

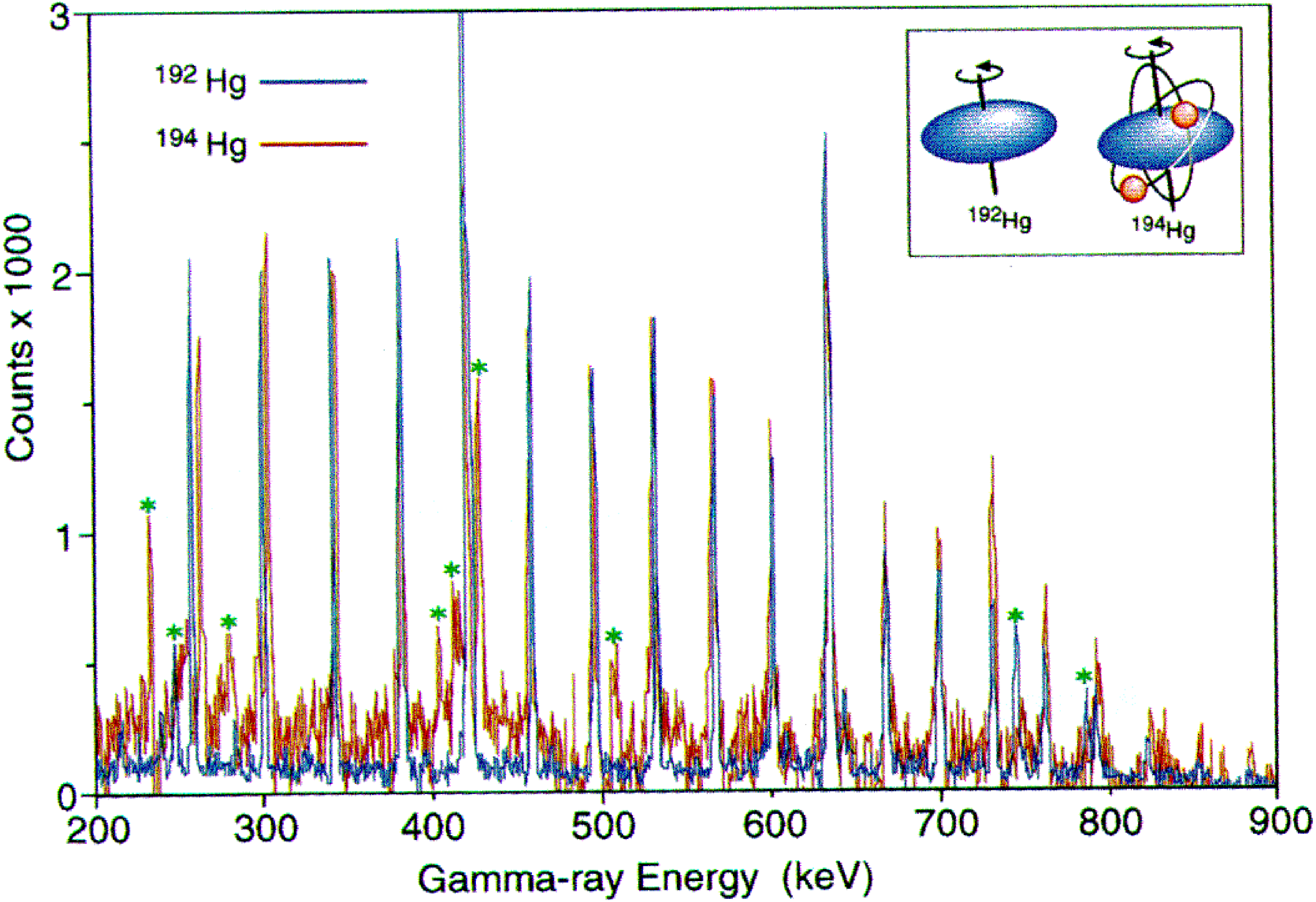
$$\Delta E^{\gamma} = E^{\gamma}_{(L+2)} - E^{\gamma}_{L} = 4\frac{\hbar^2}{\Im}$$



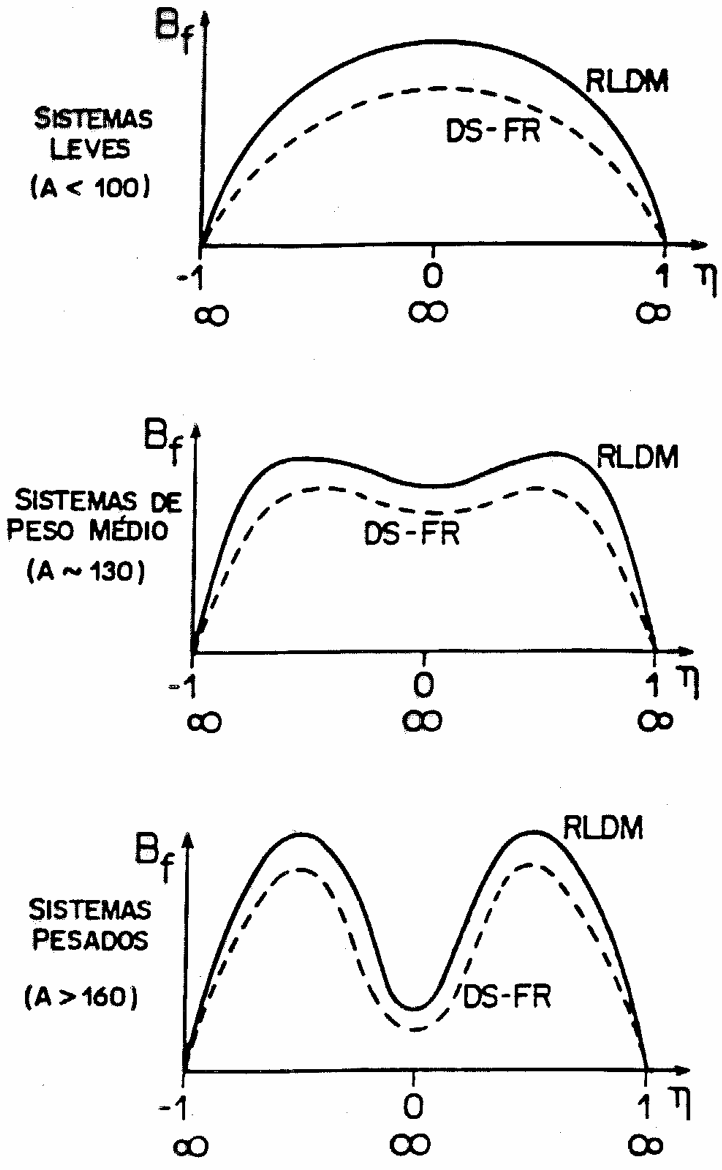

I
z


E (MeV)
36+
32+
28+
24+
20+
16+
12+
8+
4+
0+
158Er
12
10
8
6
4
2
0

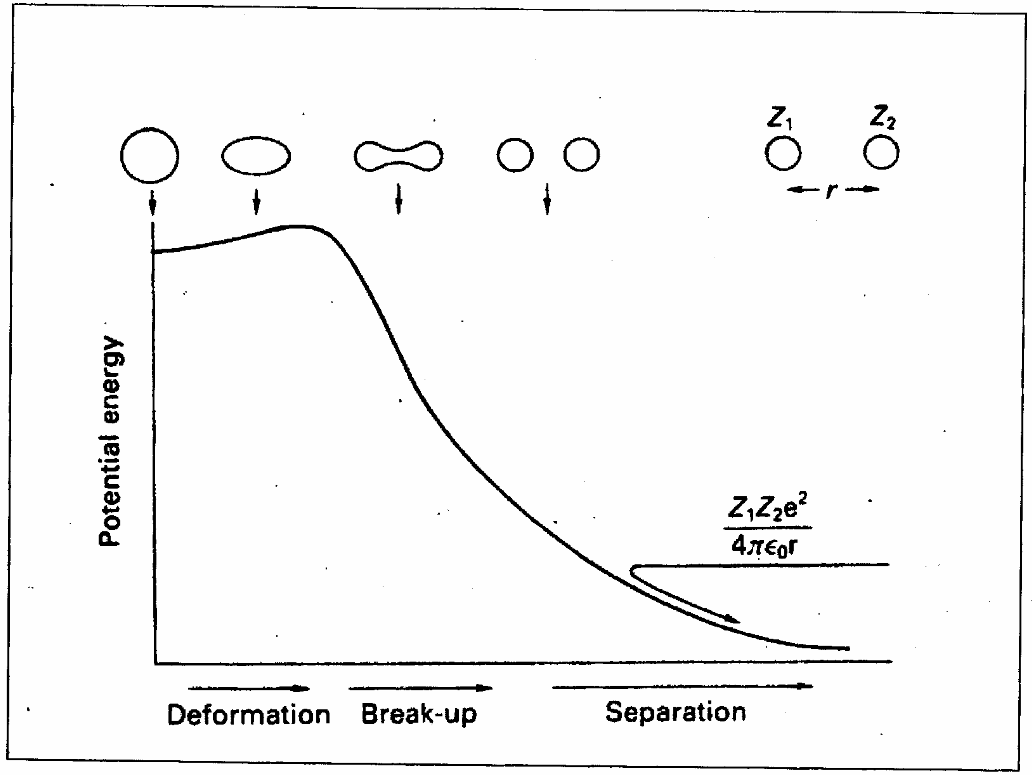

150Gd
TESSA III
16 detectors
780
814
849
888
928
970
1013
1056
1099
1144
1190
1238
1286
1337
1387
1439
1494
300
200
100
0
800
1000
1200
1400
GammaSphere
55 detectors
60000
40000
20000
0
1494
1604
(1655)
950
1150
1350
1550
Number of Counts
Energy (keV)

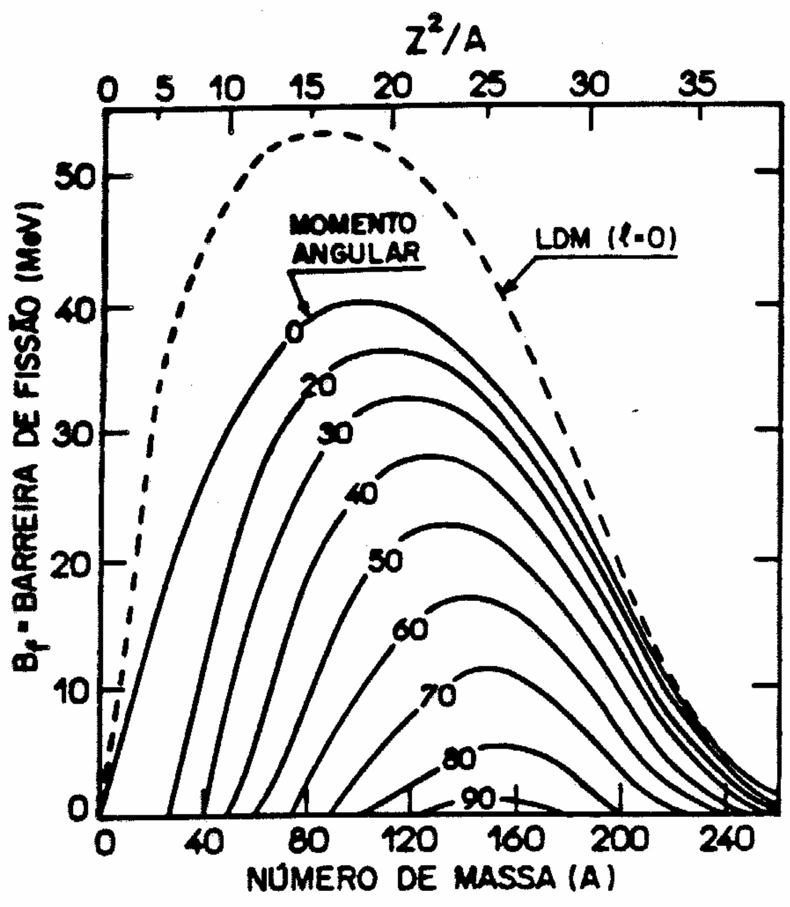


192 Hg
194 Hg
192Hg
194Hg
Counts x 1000
Gamma-ray Energy (keV)
0
1
2
3
200
300
400
500
600
700
800
900

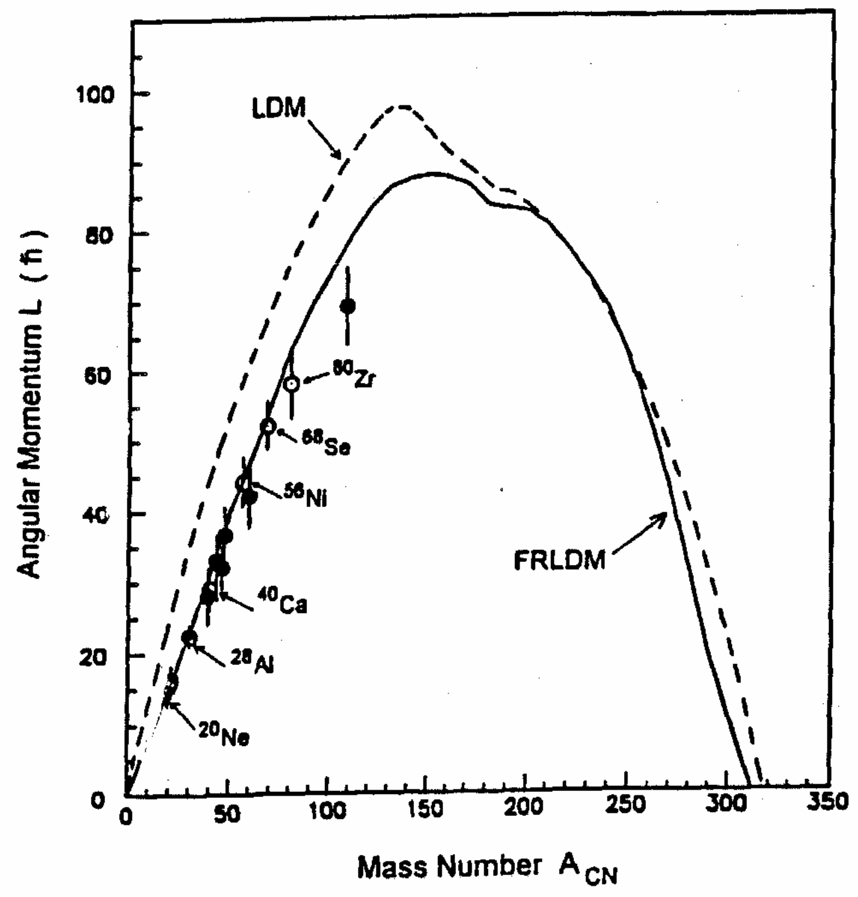


$$\eta = \frac{A_1 - A_2}{A_1 - A_2}$$



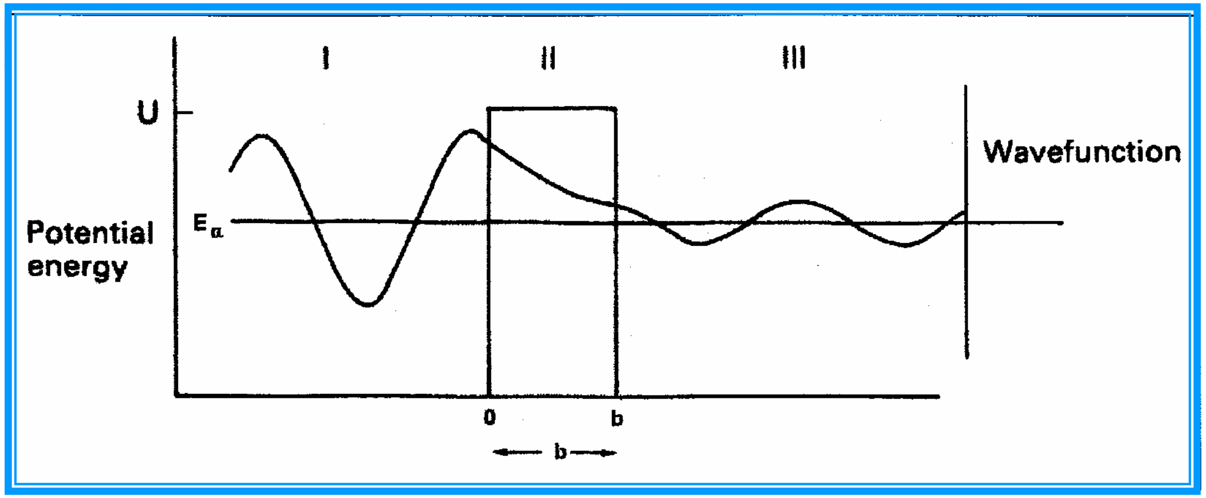


Z²/A
0 5 10 15 20 25 30 35
B_f = BARREIRA DE FISSÃO (MeV)
50
40
30
20
10
0
MOMENTO ANGULAR
LDM (ℓ=0)
0
20
30
40
50
60
70
80
90
0 40 80 120 160 200 240
NÚMERO DE MASSA (A)



Angular Momentum L (ħ)
100
80
60
40
20
0
LDM
FRLDM
⁸⁰Zr
⁶⁸Se
⁵⁶Ni
⁴⁰Ca
²⁸Al
²⁰Ne
0 50 100 150 200 250 300 350
Mass Number A_CN

Barrier Potential, $E < V_0$



$$T = |F|^2/|A|^2$$

$$T = \frac{1}{1 + \frac{1}{4}\frac{V_0^2}{E(E - V_0)}\sin^2 k_2 a}$$

$$\psi_1 = A e^{ik_1 x} + B e^{-ik_1 x}$$

$$\psi_2 = C e^{k_2 x} + D e^{-k_2 x}$$

$$\psi_3 = F e^{ik_3 x} + G e^{-ik_3 x}$$

$$k_1 = k_3 = \sqrt{2mE/\hbar^2} \text{ and } k_2 = \sqrt{2m(E - V_0)/\hbar^2}$$

$$k_2 = \sqrt{2m(V_0 - E)/\hbar^2}$$

ref. Krane

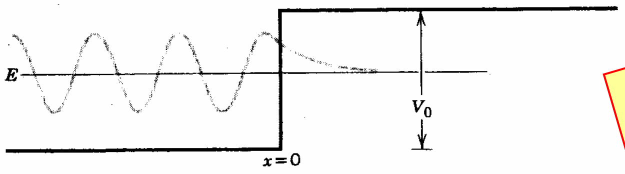


Figure 2.3 The wave function of a particle of energy E encountering a step of height V_0, for the case $E < V_0$. The wave function decreases exponentially in the classically forbidden region, where the classical kinetic energy would be negative. At $x = 0$, ψ and $d\psi/dx$ are continuous.

the classically forbidden region. All (classical) *particles* are reflected at the boundary; the quantum mechanical *wave packet*, on the other hand, can penetrate a short distance into the forbidden region. The (classical) particle is never directly *observed* in that region; since $E < V_0$, the kinetic energy would be negative in region 2. The solution is illustrated in Figure 2.3

Barrier Potential, $E > V_0$

The potential is

$$V(x) = 0 \qquad x < 0$$
$$= V_0 \qquad 0 \le x \le a \tag{2.35}$$
$$= 0 \qquad x > a$$

In the three regions 1, 2, and 3, the solutions are

$$\psi_1 = A e^{ik_1x} + B e^{-ik_1x}$$
$$\psi_2 = C e^{ik_2x} + D e^{-ik_2x} \tag{2.36}$$
$$\psi_3 = F e^{ik_3x} + G e^{-ik_3x}$$

where $k_1 = k_3 = \sqrt{2mE/\hbar^2}$ and $k_2 = \sqrt{2m(E - V_0)/\hbar^2}$.

Using the continuity conditions at $x = 0$ and at $x = a$, and assuming again that particles are incident from $x = -\infty$ (so that G can be set to zero), after considerable algebraic manipulation we can find the transmission coefficient $T = |F|^2/|A|^2$:

$$T = \frac{1}{1 + \dfrac{1}{4}\dfrac{V_0^2}{E(E - V_0)}\sin^2 k_2 a} \tag{2.37}$$

The solution is illustrated in Figure 2.4.

Barrier Potential, $E < V_0$

For this case, the ψ_1 and ψ_3 solutions are as above, but ψ_2 becomes

$$\psi_2 = C e^{k_2x} + D e^{-k_2x} \tag{2.38}$$

where now $k_2 = \sqrt{2m(V_0 - E)/\hbar^2}$. Because region 2 extends only from $x = 0$

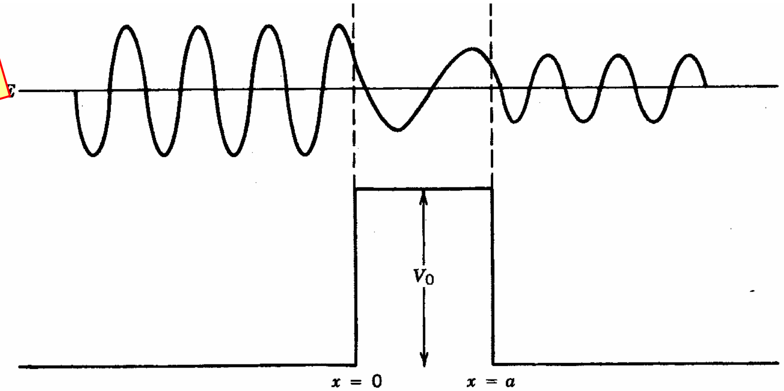


Figure 2.4 The wave function of a particle of energy $E > V_0$ encountering a barrier potential. The particle is incident from the left. The wave undergoes reflections at both boundaries, and the transmitted wave emerges with smaller amplitude.

to $x = a$, the question of an exponential solution going to infinity does not arise, so we cannot set C or D to zero.

Again, applying the boundary conditions at $x = 0$ and $x = a$ permits the solution for the transmission coefficient:

$$T = \frac{1}{1 + \dfrac{1}{4}\dfrac{V_0^2}{E(V_0 - E)}\sinh^2 k_2 a} \tag{2.39}$$

Classically, we would expect $T = 0$—the particle is not permitted to enter the forbidden region where it would have negative kinetic energy. The quantum wave can penetrate the barrier and give a nonzero probability to find the particle beyond the barrier. The solution is illustrated in Figure 2.5.

This phenomenon of *barrier penetration* or quantum mechanical *tunneling* has important applications in nuclear physics, especially in the theory of α decay, which we discuss in Chapter 8.

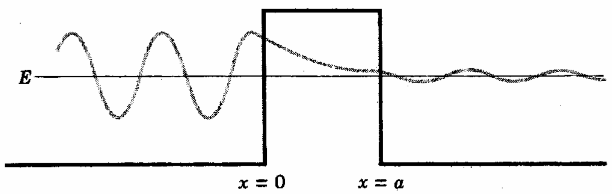
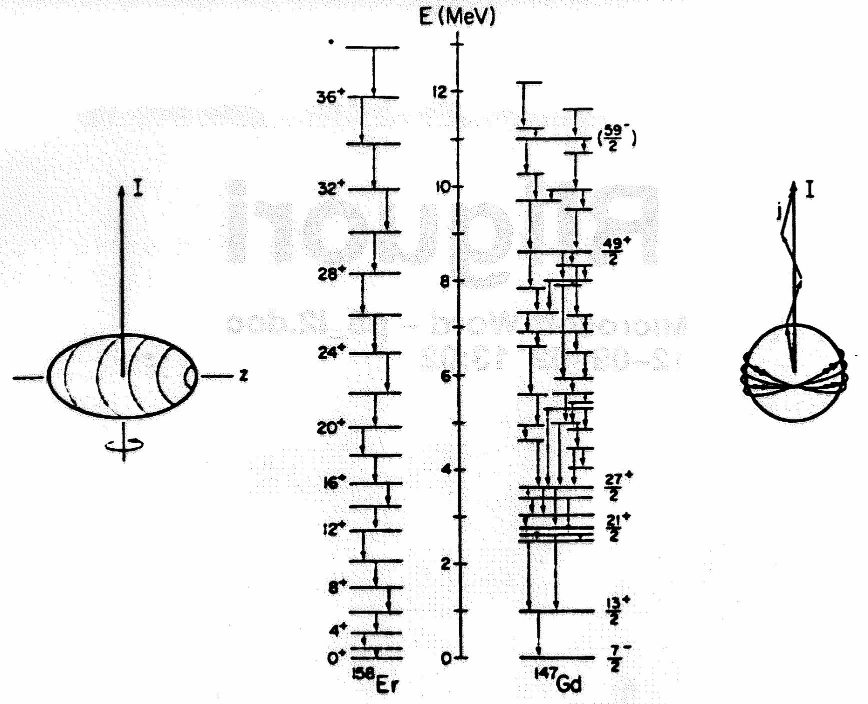
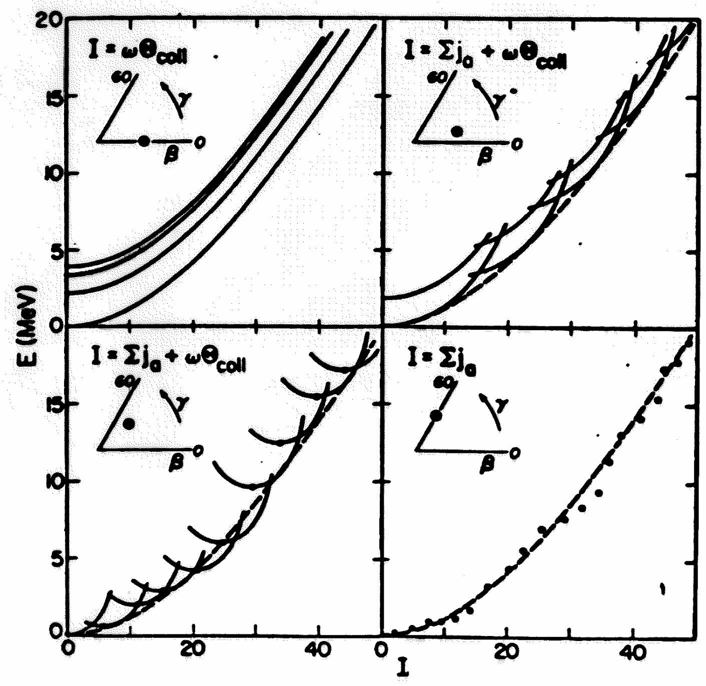
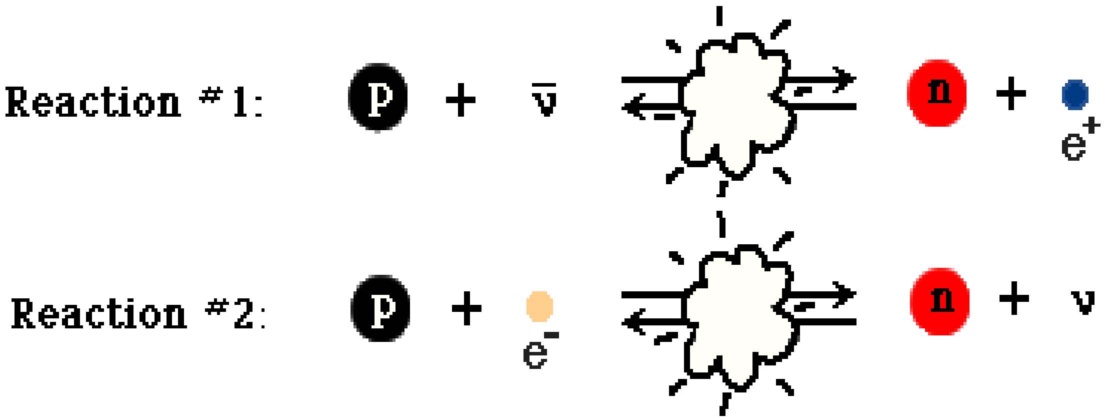


Figure 2.5 The wave function of a particle of energy $E < V_0$ encountering a barrier potential (the particle would be incident from the left in the figure). The wavelength is the same on both sides of the barrier, but the amplitude beyond the barrier is much less than the original amplitude. The particle can never be observed, inside the barrier (where it would have negative kinetic energy) but it can be observed *beyond* the barrier.

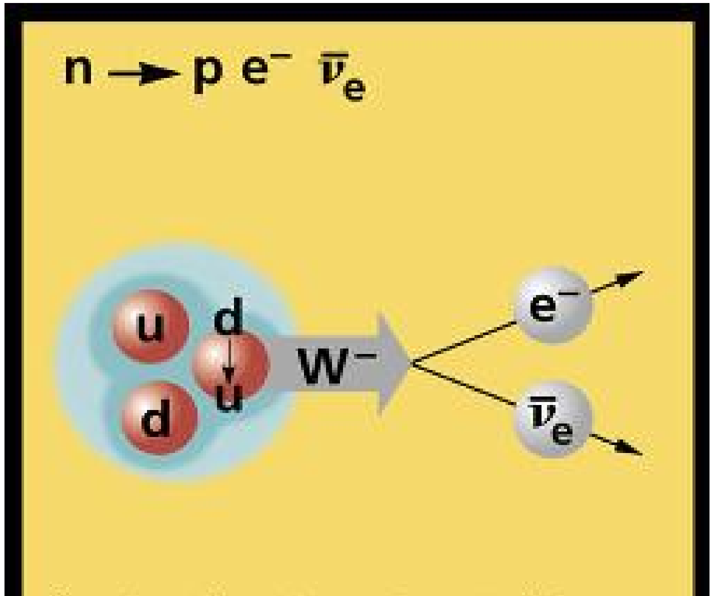

E (MeV)
I
z
j
36+
32+
28+
24+
20+
16+
12+
8+
4+
0+
158Er
147Gd
(59-/2)
49+/2
27+/2
21+/2
13+/2
7-/2
0
2
4
6
8
10
12


I = ωΘcoll
I = Σja + ωΘcoll
I = Σja + ωΘcoll
I = Σja
γ
β
60
0
E (MeV)
I
0
5
10
15
20
0
20
40

proton/neutron conversions



(The double arrows indicate these reactions go both ways.)



A neutron decays to a proton, an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β decay.

BOSONS

Unified Electroweak spin = 1

Name	Mass GeV/c²	Electric charge
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c²	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c²	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3